Wavelet and Spectral Analysis of Some Selected Problems in Reactor Diagnostics

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Abstract

Both spectral and wavelet analysis were successfully used in various diagnostic problems involving non-stationary core processes in nuclear power reactors. Three different problems were treated: two-phase flow identification, detector tube impacting and core-barrel vibrations. The first two problems are of non-stationary nature, whereas the last one is not.

In the first problem, neutron radiographic and visible light images of four different vertical two-phase flow regimes, bubbly, slug, churn and annular flow, were analysed and classified with a neuro-wavelet algorithm. The algorithm consists of a wavelet part, using the 2-D discrete wavelet transform and of an artificial neural network. It classifies the different flow regimes with up to 99% efficiency.

Detector tubes in a Boiling Water Reactor may execute vibrations and may also impact on nearby fuel-assemblies. Signals from in-core neutron detectors in Ringhals-1 were analysed, for detection of impacting, with both a classical spectral method and wavelet-based methods. The wavelet methods include both the discrete and the continuous 1-D wavelet transform. It was found that there is agreement between the different methods as well as with visual inspections made during the outage at the plant. However, the wavelet technique has the advantage that it does not require expert judgement for the interpretation of the analysis.

In the last part two analytical calculations of the neutron noise, induced by shell-mode core-barrel vibrations, were carried out. The results are in good agreement with calculations from a numerical simulator. An out-of-phase behaviour between in-core and ex-core positions was found, which is in agreement with earlier measurements from the Pressurised Water Reactor Ringhals-3. The results from these calculations are planned to be used when diagnosing the shell-mode core-barrel vibrations in an operating plant.

Keywords: noise diagnostics, wavelet analysis, detector tube impacting, two-phase flow, core-barrel vibrations
This thesis consists of an introduction and the following papers:


Scientific publications related to the licentiate topic but not included in this thesis:


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1. Introduction

Noise diagnostics has been used in Nuclear Power Plants (NPPs) since the earliest days of reactor physics [1-4], beginning fifty years ago with the start of the first reactor. Different methods have been developed during the decades up till now and still there is a lot of development going on. This thesis is a continuation of the noise diagnostic work at the Department of Reactor Physics, Chalmers University of Technology. Then, what is noise diagnostics?

In ordinary life, noise is considered to be something unwanted or disturbing, the noise in radio broadcasting or disturbing noise which can occur when speaking in a cell-phone. In order to get a clear, noise-free signal, the noise is often filtered away. However, in noise diagnostics it is the other way around, the static signal is filtered out and the noise is considered as the important part of the signal. Thus, the noise is extracted rather than filtered away. What is the use of the noise? Take, again, the cell-phone as an example. When calling to someone, there may be some disturbances (noise) in the voice transmitted over the phone. Based on the noise it could be possible to draw the conclusion that the person you are calling for example is driving a car, which is an example of a simple form of noise diagnostics. Hence, the noise is used to draw conclusions about a system, which can be very useful.

In reactor noise diagnostics, the noise in signals from detectors placed inside and outside the reactor core are used to diagnose the reactor. Compared to the simple cell-phone example the situation is somewhat different when diagnosing a reactor. First, the magnitude of the noise can be a couple of powers of ten lower compared to the signal itself. Second, it is possible to have more than one noise-source. Third, it is not always easy to get a simple response; you can not simply ask a reactor whether it is driving a car or not! Rather, continuing with the same example, it is like finding out in which of New York’s thousands of streets the car is in, what speed it has etc.

The purpose of reactor diagnostics is mainly to have control over the reactor and make sure everything is working properly. However, if something starts deteriorating, the on-line diagnostics of the reactor should detect the error and if necessary alert the operator, who in return may initiate a shut-down. The off-line diagnostics of the reactor means mainly to investigate the behaviour of the reactor and to give information about the reactor status, e.g. find components that may need maintenance or understand trends, unexpected phenomena etc. Hence, the diagnostics must be as reliable as possible. One does not want to stop the reactor by false alarm or maintain a working component, since, it is time consuming to restart the reactor and, of course, there is a loss of money if the reactor is not operating. Hence, it is important to understand and develop noise diagnostic methods, which are reliable. The methods developed in this thesis are not for on-line use in power producing nuclear plants, but more of an understanding level of the processes in a reactor and for off-line use.

At the Department of Reactor Physics, Chalmers University of Technology, reactor noise diagnostics have been used for different diagnostic tasks [5-26], such as detection of detector tube impacting, identification of core boundary (barrel) vibrations and Moderator Temperature Coefficient (MTC) determination. There are two ongoing research projects on noise diagnostics, one in cooperation with the Swedish NPP Ringhals [10-17], and one in cooperation with the Swedish Nuclear Power Inspectorate (SKI) [18-26].
In this thesis noise diagnostics is applied within the research areas of core-barrel vibrations, two-phase flow identification and detector tube impacting. One of the major goals of this thesis is to include wavelet analysis in the noise diagnostic work. Wavelets are especially suited for analysing non-stationary processes, e.g. intermittent signals. Hence, the focus of the noise diagnostics performed in this thesis is on non-stationary processes.

Since wavelets are a relatively new tool in reactor physics, a short introduction to wavelet theory is given in Section 2. The outline for the rest of the thesis basically follows the order of the appended papers. The work done on two-phase flow identification, which is described in papers I and II, is summarised in Section 3. Section 4 is a summary of the work with detector tube impacting found in paper III. Section 5 is a summary of the work with core-barrel vibrations described in papers IV and V.
2. Wavelets

One goal of this thesis and the research within the project is to use wavelet techniques in reactor noise diagnostics. Wavelets are still considered to be a new field in signal processing, even though they have been in use for almost two decades since they were first introduced in the mid-80s. The real development started in the early 90s [27], and has continued ever since by the use of wavelets in different scientific areas such as fluid dynamics, medicine, finance, physics and geophysics [28]. This Section gives a very short introduction to wavelets and their application. A more mathematical detailed explanation can be found in [27-31].

2.1 Time and frequency

The classical Fourier transform can be used to map a time signal into the frequency domain, as illustrated in Fig. 1. The Fourier transform can only be applied on stationary signals, where there are no changes in frequency over the time interval of interest. However, if the signal is non-stationary, the Fourier transform cannot be used. In that case a windowing of the signal can be done, using the so-called windowed Fourier transform or Short Time Fourier Transform (STFT). The STFT maps a time signal into a two-dimensional signal of both time and frequency. Hence, it is possible to get information about both when and at what frequency a certain event occurs. However, there is one drawback with the STFT, namely that the frequency resolution is the same for all frequencies. Often there is a need for better resolution in time at higher frequencies. The next step is to construct a tool which can map a time signal into time and frequency but with different resolution for different frequencies. The wavelet transform is able to cope with this requirement, even though it is mapping the signal into time-scale or time-level rather than time-frequency, but there is a connection between scale, level and frequency. However, the better resolution at high frequencies in time is achieved by reducing the resolution in frequency. The resolution in time becomes poorer at lower frequencies as can be seen in Fig. 1.

The Fourier transform uses an infinitely long sinusodial function as the analysing tool, hence it has no time resolution. On the other hand the wavelet transform uses a small, localized wave function (see Fig. 2) as the analysing tool, hence the name wavelet. The use of localised functions makes the wavelet transform well suited for analysing non-stationary signals such as transients and intermittent signals. The wavelet displayed in Fig. 2 is the so-called *Mexican hat*, which has the characteristic features of a wavelet. However, this wavelet will not be used in the rest of the Section since it does not have compact support and is not orthogonal. Without going into details, this means that it can not be used when performing the discrete wavelet transform.
In the rest the Daubechies 4 ($db4$) wavelet will be used instead, since it has compact support and is orthogonal [27].

![mexican hat wavelet](image)

**Fig. 2.** The Mexican hat wavelet has the form of a small localized wave, hence the name wavelet. The dotted curve is a cosine function with the same frequency as the centre frequency of the Mexican hat wavelet.

### 2.2 One-dimensional wavelet transform

It is possible to use the wavelet transform on one- or two-dimensional data. In this subsection the one-dimensional transform is described and in the next a short description of the two-dimensional wavelet transform is given.

There are two ways of performing the one-dimensional transform: continuous with continuous scales or levels (frequencies), or discrete with discrete scales or levels (frequencies). In this thesis the focus is on the discrete transform, hence this will be described in more details. Although part of this thesis touches upon the continuous wavelet transform, it will not be described in this Section.

The wavelet transform is based on a so-called mother-wavelet, $\Psi$, which is dilated and translated (see Fig. 3), with the parameters $a$ and $b$.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$  \hspace{1cm} (1)

At each scale $a$ there is a characteristic frequency, $F_a$, which can be calculated through the so-called centre-frequency of the mother-wavelet, $F_c$, and the sampling period, $\Delta t$, of the analysed signal:

$$F_a = \frac{F_c}{a\Delta t}$$  \hspace{1cm} (2)

The centre-frequency is the characteristic frequency of the mother-wavelet, see Fig. 2. One way of choosing the parameters in (1) are, $a = a_0^m$ and $b = nb_0a_0^m$ where $m$ is called the
level. The most common choice of \( a_0 \) and \( b_0 \) are 2 and 1 respectively. This gives the discrete one-dimensional wavelet transform of the signal \( x(t) \) as:

\[
T_{m,n} = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{2^m}} \psi(2^{-m} t - n) dt
\]  

(3)

Here \( T \) is called the detailed wavelet coefficients. They contain information about the details of \( x \). The coefficients \( T_{m,n} \) are calculated by translating and dilating the mother wavelet along the signal, illustrated in Fig. 3, and performing the integration at each step. The transform is performed at each discrete level \( m \).

With each wavelet there is an associated scaling-function, \( \phi \). The wavelet and the scaling functions are orthogonal and have the following relation:

\[
\psi(t) = \sum_{k} (-1)^k c_{1-k} \phi(2t - k)
\]  

(4)

Here \( c \) is a scaling coefficient. The scaling functions can be used to calculate approximation coefficients of the signal, \( x(t) \), in the same way as the calculation of the detail coefficients:

\[
S_{m,n} = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{2^m}} \phi(2^{-m} t - n) dt
\]  

(5)
The approximation coefficients contain information about the mean behaviour of the signal. With the use of both the detail coefficients and the approximation coefficients the signal can be reconstructed as:

\[ x(t) = x_{m_0}(t) + \sum_{m = -\infty}^{m_0} d_m(t) \]  \hspace{1cm} (6)

where the detail of the signal, at level \( m \), is defined as:

\[ d_m(t) = \sum_{n = -\infty}^{\infty} T_{m,n} \frac{1}{\sqrt{2^m}} \psi(2^{-m} t - n) \]  \hspace{1cm} (7)

and the approximation of the signal at level, \( m_0 \), is:

\[ x_{m_0}(t) = \sum_{n = -\infty}^{\infty} S_{m_0,n} \frac{1}{\sqrt{2^{m_0}}} \phi(2^{-m_0} t - n) \]  \hspace{1cm} (8)

If the input signal, \( x_0 \), is discrete and of finite length, e.g. \( N = 2^M \), as it normally is when dealing with measurements, \( m \) and \( n \) are also finite and it is possible to rewrite the above formulae as follows:

\[ x_0(t) = x_M(t) + \sum_{m = 1}^{M} d_m(t) \]
\[ x_M(t) = S_{M,n} \frac{1}{\sqrt{2^M}} \phi(2^{-M} t - n) \]  \hspace{1cm} (9)
\[ d_m(t) = \sum_{n = 0}^{2^M - n - 1} T_{m,n} \frac{1}{\sqrt{2^m}} \psi(2^{-m} t - n) \]

Here \( x_M \) is the mean of the signal. From this it is possible to get a relation between approximation and detail at one level and the approximation at the next level:

\[ x_{m-1}(t) = x_m(t) + d_m(t) \]  \hspace{1cm} (10)

With this choice of \( a \) and \( b \) the decomposition of the signal into details and approximation is called the wavelet multiresolution analysis. From the multiresolution it is clear that the wavelets, used in the discrete transform, consists of an orthogonal set of basis functions in which an arbitrary function can be expanded.

As an example of the multiresolution analysis take a chirp signal, shown at the top in Fig. 4 a), with some white noise added. It is decomposed into details and approximations at different scales using the \( db4 \) wavelet in the MatLab Wavelet Toolbox [31]. The details at each level represent the content of the signal at that level (frequency). Hence it is possible to extract
information about both when and at what frequency a certain event happens. As expected, the noise is present in the lowest level (highest frequency) details 1-3, see Fig. 4 b). The high frequency components of the regular chirp signal are visible at the middle levels, 5-6, to the right in Fig. 4 b). The low frequency components of the regular chirp signal are visible to the left at higher levels, 7-8, in the same figure. An almost noise free chirp signal is visible at approximation level 4, \(x_4(t)\) in Fig. 4 a).

![Fig. 4. Approximation \(x_m\), a), and details \(d_m\), b), at ten different levels of the wavelet transform using the \(db4\) wavelet. The original signal \(x_0\) is at the top in both plots.](image)

The detail coefficients \(T_{m,n}\) contain the same information as the reconstructed details, \(d_m\), and the same is valid for the approximation coefficients \(S_{m,n}\) and the reconstructed approximation, \(x_m\). Hence it is possible to use either the coefficients or the reconstructed approximation and details when analysing the signal. The advantage with the coefficients is that the size of them are decreasing with a factor of two at each level, compared with the reconstructed signals which have the same size as the original signal.

From Fig. 4 it is possible to draw the conclusion that wavelets can be used to denoise a noisy signal. By taking the approximation at level 4 of the signal, an almost noise-free chirp signal can be obtained. However, all information from detail level 1-4 is neglected, when using this approximation. One way of performing a better denoising is to use some information from the lowest levels (highest frequencies). This can be done by thresholding the detail coefficients, in this example details at level 1-4. When reconstructing the signal, the approximation and the thresholded detail coefficients are used to calculate a denoised representation of the signal. Also it is seen that the noise can be extracted through the details. By using the three first levels of details in Fig. 4 b) it is possible to almost completely extract the noise. An application where this could be of use is the possibility to extract short intermittent parts of a signal, so-called spikes.

The one-dimensional wavelet transform (multiresolution) will be used in Section 4.

### 2.3 Two-dimensional wavelet transform

If the input data are two-dimensional, i.e. an image, it is possible to use a two-dimensional wavelet transform. The principles of the transformation are the same as for the one-dimensional transform, but instead of having one detail at each level there are three.
information in each of the three details are from three directions of the two-dimensional input data, horizontal, vertical and diagonal details. The first equation in (9) can then be expressed as

\[ x_0(x, y) = x_M(x, y) + \sum_{m=1}^{M} d^h_m(x, y) + d^v_m(x, y) + d^d_m(x, y) \] (11)

This is the reconstruction for the two-dimensional discrete wavelet transform. The details, the detail coefficients, the approximations and the approximation coefficients are calculated in the same way as for the one-dimensional transform, except that different two-dimensional wavelets are used for the different directions and a two-dimensional scaling function is used. All the two-dimensional wavelet and scaling functions can be calculated from the one-dimensional ones.

\[ \psi^h(x, y) = \phi(x)\psi(y) \]
\[ \psi^v(x, y) = \psi(x)\phi(y) \]
\[ \psi^d(x, y) = \psi(x)\psi(y) \]
\[ \phi(x, y) = \phi(x)\phi(y) \] (12)

Depicted in Fig. 5 b)-e) are the reconstructed approximation and details at the first level (highest frequency) of a two-dimensional wavelet transform, using the db4 wavelet, on the image in Fig. 5 a). In the horizontal detail c), the horizontal high frequency parts (sharp transitions) are clearly visible, e.g. the line in the middle. In the same way the vertical high frequency parts (the trees) are clearly visible in d), the vertical details, as expected. There are no clear features in the diagonal details e), since the original images have no sharp diagonal transitions. The two-dimensional transform can be used to characterise an image by identifying different features of the image at different details levels (scales) and/or different directions. This is used in Section 3.

Another feature of the wavelet transform, which is not used in this thesis, is the possibility to compress images. The differences between the original image, a), and the first level approximation, b), are hardly noticeable. However, the approximation contains only 25% the amount of data as the original image. Hence, it can be used as a compressed version.

For further information on wavelet theory and applications the interested reader is referred to [27-31]. There all the mathematical details, which are left out in this thesis, can be found.
Fig. 5. Approximation, b), and details, c)-e), from a two-dimensional wavelet transform, using db4 wavelet, at the first level. The original image is depicted in a).
3. Two-phase flow identification

The first application of wavelet techniques in this thesis is in the area of two-phase flow identification. It is very important to classify the different flow regimes in a reactor, since they have quite different flow properties. Before using a flow equation, the regime must be determined in order for the right expression to be chosen for e.g. the interfacial shear coefficient or some other coefficients like the heat transfer coefficient. The task at hand is to classify two-phase flow regimes with image analysis.

The approach in this Section is to use wavelets to pre-process flow images and then extract statistical features to use as inputs to an Artificial Neural Network (ANN), see Fig. 6. The use of dynamic images as the “signal” from the flow means that the method is non-intrusive. Both non-intrusive methods, wavelets and ANN have been used previously when classifying two-phase flow regimes [32-37]. However, they have not been used in combination before. A more detailed description of the two-phase flow identification is found in paper I and II.

In nuclear technology the two-phase flow is water and vapour which flow within the core of a boiling water reactor. In vertical two-phase flow there are four main regimes, bubbly, slug, churn and annular flow. Bubbly flow is the flow of dispersed vapour in continuous liquid, with small bubbles of vapour in the water. In slug flow the bubbles of vapour have formed larger regions, with a size of approximately the size of the pipe diameter. If even more vapour is present in the pipe the bubbles break and there is an unstable regime of liquid mix with vapour, churn flow. In the last type, annular flow, the pipe is almost filled with vapour, only a thin part, close to the wall, contains liquid. The geometrical structure of the regimes is very different, but at the same time, it is difficult to express this difference in quantitative terms. This is why two-phase flow identification is difficult to perform with algorithmic methods.

3.1 Flow images

Two different sets of flow images have been analysed. The first set included dynamic neutron radiographic images of two-phase flow within a metallic water loop, see Fig. 7 a). This experiment was performed at the Kyoto University Reactor Research Institute (KURRI) [38]. By continuously increasing the heating of the water in the loop, all four flow regions were created in sequence. One drawback with these images is that they had to be converted into digital format since the recording was available on an analogue VHS in NTSC format. This conversion somewhat increases the noise in the images.

The second set of images, recorded at our department with the use of an ordinary digital camcorder, used visible light instead of neutron radiography. In the experiment the two-phase flow was simulated by injecting air into a thin rectangular pipe filled with coloured water, see Fig. 7 b). Due to the simple set-up it was only possible to simulate bubbly and slug flow. On the
other hand, a much better image quality was achieved as compared to the neutron radiographic images.

![Fig. 7.](image)

**Fig. 7.** a) Images of the four different flow regimes using neutron radiography, flicker noisy images.

b) Images of bubbly and slug flow regimes using visible light and coloured water, almost noise-free images

### 3.2 Wavelet pre-processing

Before extracting inputs for the ANN, the images (2-D matrix with gray scale pixel intensity) were pre-processed with the two-dimensional wavelet transform. Wavelets are suitable to highlight features of different length scales (frequencies) in an image. It is also possible to extract information about features in different directions in an image, see Fig. 5 (Section 2.3). The different flow regimes are assumed to have different features at different length scales and in different directions. The idea is to use some features of the transformed wavelet coefficients which are characteristic for each regime. One possible feature is the energy content of the detail coefficients. In this case the first level of detail coefficients are considered and one way of expressing the energy content is given in [28] as:

\[
E_1^x = \sum_i \left| T_1^x, i \right|^2
\]  

Here \(x\) stands for the different directions (horizontal, vertical and diagonal) in the two-dimensional transform, see Section 2.3 for details. The energy feature was used to characterise the visible light images. Unfortunately, using the energy feature did not work in the case of the radiographic images, probably due to the flicker noise. Instead, the mean value of the first level approximation coefficients was used. Another possible feature, which could be used, is the variance of the very same coefficients. Hence, one value for the neutron radiographic images, the variance of the approximation coefficients, and three values for the visible light images, one for the details coefficients in each direction are also used. This gives a total of two features extracted for the radiographic images and six features for the visible light images. These features are, in the next step of the classification process, used as inputs for an ANN.

### 3.3 The classification algorithm

The classification task is solved by using an ANN with the wavelet pre-processed features as inputs. The principle of an ANN is to have a set of inputs with known output values. The ANN is fed with these inputs which propagate through the nodes of the network. The output is
compared with the known output values and some error parameters are calculated which are used to change the nodes. There are different ways in changing the nodes, i.e. different training procedures. The training is repeated until some pre-defined minimal criterion of the error is reached. Each repetition is called an epoch. After the training it is possible to feed the ANN with inputs whose outputs are to be determined. A properly trained ANN will generate the correct output values. For the two-phase flow classification the right flow regime type is searched.

The ANN used in this thesis is constructed from the Neural Network Toolbox in MatLab. Different types of networks and training algorithms were tested and by trial and error the best combination was selected for the task at hand. A feed-forward network with an input layer, an output layer and one hidden layer trained with the resilient backpropagation (BP) algorithm was found to be the most effective one. The number of input nodes used depends on which type of flow images is used. Two input nodes were used for the radiographic images and six for the visible light images. The number of nodes in the output layer also depends on the images, one output for each flow regime that is classified. Hence, four output nodes were used for the radiographic ones and two for the visible light images. The \textit{log-sigmoid} transfer function was used for the output layer, giving values between 0 and 1. For both types of images 40 hidden nodes with \textit{tan-sigmoid} transfer functions were used. The training target values were set to 0.9 for the correct regime and 0.1 for the other outputs.

The input data consist of 200 images from each of the flow regimes in the case with neutron radiography and 75 images from each of the two regimes in case of the visible light. A 5-fold cross-validation over the training data was used, i.e. 1/5 of the input data was used as a test set to verify the classification success of the network trained with the remaining 4/5 of the inputs. This was repeated five times by using different images in the test set. All images were used only one time in the test set. When classifying the test set, the outputs were thresholded in order to get either 1 for the right flow or 0 for the wrong ones. If more than one output is 1 or none is 1 the image is classified as unknown flow regime.

3.4 Results from the classification

To investigate the advantage with the wavelet pre-processing, the mean value and the variance from the raw image pixel intensity were also fed in to the network, and the results were compared to those with the wavelet pre-processed input data. Six different discrete wavelets were used for the pre-processing, \textit{Haar}, \textit{Daubechies 8}, \textit{Coiflet 4}, \textit{Symmlet 6} and \textit{Biorthogonal 3.1} all available in the Wavelet Toolbox in MatLab. In the case of the noisy radiographic images, the success ratio of the classification was around 95\% for the test set of images for all the different wavelets and the same for the raw data input. In Fig. 8 the result for the input pre-processed with the \textit{Daubechies 8} wavelet in shown. Hence, there is no advantage, from the point of view of success ratio, in pre-processing the images. When using the visible light images the success ratio was even higher, around 99\%, for both the pre-processed and the raw inputs. However, the number of epochs used during the training procedure is reduced with a factor of 100 when using wavelet pre-processing. For the raw data input the maximum number of epochs, set to 30 000, was always reached before the target value of the Mean Square Error (MSE), set to $10^{-3}$ was reached. In the pre-processed case the MSE target was reached within approximately 300 training epochs. Hence, the use of wavelet pre-processing has large advantages from the practical point of view.
Also worth mentioning is that 100% of the annular flow images were classified correctly in the neutron radiographic case. As expected, the slug flow and the churn flow were the ones most likely to be mixed up, i.e. classified wrong or as unknown.

The next step in the project will be to analyse neutron radiographic images, recorded in digital format. Hopefully, this will give images with the same quality as the visible light images, but with all four flow regimes available. It is expected that in the end this will lead to better performance of the wavelet pre-processing when used on all four regimes.

**Fig. 8.** Classification ratio of the neutron radiographic images using a threshold of 0.5 after the ANN. A total of 200 images from each regime were classified and the average success ratio was 95%.
4. Detector tube impacting

Methods for detection of detector tube impacting in Boiling Water Reactors (BWRs) with noise diagnostic methods, applied to signals from neutron detectors, have been used for a long time. The first methods used were based on classical Fourier analysis [3, 39]. In these methods the Auto Power Spectral Density (APSD), coherence and phase curves are used to identify impacting tubes. Typically, a broadening of the eigenfrequency peak in the APSD and/or a distorted phase curve give information about an impacting tube. More about the spectral methods will be mentioned below. At our Department a new method, based on wavelet analysis, has been developed. First, simulations and measurements from the Swedish nuclear power plant Barsebäck-2 were analysed with both spectral and wavelet methods [5-6]. Later, measurements taken from Oskarshamn-2 were analysed and the result was compared with visual inspections made before the analysis [24-25, 40]. Between the two different analyses the wavelet algorithm was modified. In this Section a continuation of these analyses is summarised. For a detailed description see paper III. This time, measurements from Ringhals-1 are analysed both with the traditional spectral methods and the new wavelet based method. Recently the continuous wavelet transform has also been applied to the same problem, and it is also described briefly in the thesis.

4.1 Physical model

The task is to identify detector tubes which not only vibrate but also impact on the neighbouring fuel assemblies. If a detector tube hits a fuel assembly it can damage the fuel box which may cause also damage to the fuel cladding. Any such event must be avoided in an operating plant.

![Diagram of detector tube in a BWR core with surrounding fuel assemblies](image)

Fig. 9. Illustration of a detector tube in a BWR core with surrounding fuel assemblies, three out of four shown. Some typical data of interest are also shown.

Fig. 9 shows a general outline of the physical setup of a detector tube together with the surrounding fuel assemblies. The vibrations arise from the strong flow of coolant water in the reactor and the fact that the detector tubes, which are roughly four meters long, are fixed only in their ends. The eigenfrequency is around 2 Hz, [3]. If the vibration is large enough, the tube
may impact on the nearby fuel assemblies, which in return will execute a short, damped oscillation after each hit, with an eigenfrequency of 10 to 20 Hz, [3]. Detectors are placed at four different axial levels in the tube. From the Ringhals-1 measurements, analysed here, signals from two of the four detectors in each tube were available. There are 36 detector tubes evenly distributed in the core giving a total of 72 signals in the Ringhals-1 case.

The signals from vibrating and impacting detector tubes are assumed to contain three parts. First a global part with some broad-band noise, \( N(t) \). The second part is a regularly oscillating part from the detector tube vibration, \( S(t) \), and thirdly an intermittent part due to the vibration of the fuel-assembly, \( T(t) \). The intermittent structure is due to the damped, randomly occurring vibration of the fuel-assembly, see Fig. 10. In general, the amplitude or the root mean square value of \( T(t) \) is much smaller than that of \( N(t)+S(t) \). Therefore, in a spectral analysis, the effect of \( T(t) \) is not visible. Hence, the classical spectral method focuses on the vibration of the detector tubes, whereas the wavelet method tries to identify the intermittent signal from the fuel assembly vibrations. Based on this fact the sampling frequency used in the wavelet method needs to be higher, since the eigenfrequency of the fuel assemblies is higher than for the detector tubes.

\[
\Phi(t) = N(t) + S(t) + T(t)
\]

\[\text{Flux gradient}\]
\[\text{S(t): Detector string noise (Stationary)}\]
\[\text{N(t): Global neutron noise}\]
\[\text{T(t): Fuel box noise (Transient)}\]

Fig. 10. Schematic view of the signal, \( S(t) \), from a detector vibrating in a flux gradient with background noise, \( N(t) \), and fuel assembly impacting, \( T(t) \).

4.2 Analysing the measurements

As mentioned above, measurements from the BWR Ringhals-1 were analysed. In cooperation with plant personnel two different measurements were made. Both measurements were taken during full power (109%) and full core flow (~11 000 kg/s). The first measurement was carried out on 6th of September, 2002. In the rest of this thesis this measurement is referred to as measurement 1. The second measurement was made on 27th of October, 2003, referred to as measurement 2.

In measurement 1 the sampling frequency was too low, 12.5 Hz, for the wavelet method to be performed and the measurement time was approximately 11 min. For measurement 2, on the other hand, the sampling frequency was raised to 64 Hz to better use the information from
the fuel assembly vibrations (10-20Hz). This time the duration of the measurement was 5 min.
At the moment a third measurement is planned with even higher sampling frequency, 200 Hz.

4.2.1 Measurement 1

Since measurement 1 was not suited for wavelet analysis, the judgment on which tubes that may impact on the fuel assemblies is based mainly on the spectral method. The spectral method is based on the features of the Auto Power Spectral Density, APSD, the coherence, and the phase between two detectors in the same tube. The following four criteria are used to classify a detector tube as an impacting one [5]. First, broadening of the eigenfrequency peak in the APSD compared to a vibrating but non-impacting tube. Second, multiple peaks in the APSD, especially at the double eigenfrequency. Third, high coherence at the eigenfrequency, and finally, distorted (zero) phase over a large frequency range. The decision of whether a tube is impacting or not is made by qualitatively taking all four criterions into account. This is more or less a relative method. The idea with the wavelet method is to calculate a so-called Impact Rate index (IR-index). The IR-index gives the number of intermittent signals due to the fuel assembly vibration. A high value means severe impacting. By using the IR-index, there is no need for an expert judgment which is the case for the spectral method. Hence, the wavelet method is an absolute method.

As mentioned above the detector signals are assumed to consist of three parts:

\[ \phi(t) = N(t) + S(t) + T(t) \]  

where \( S(t) \) is analysed in the spectral case, described above, and \( T(t) \) is extracted in the wavelet method which will be described below.

To reduce the high frequency noise present in the signal, a wavelet denoising is performed. The denoising is done by applying a level-dependent threshold to each of the detailed coefficients, \( T_{m,n} \) as described in Section 2, in the multiresolution analysis. The wavelet decomposition is made down to a level, \( M \), corresponding to the eigenfrequency of the fuel assemblies (10-20 Hz). The denoised signal is then reconstructed by using the thresholded detailed coefficients in (9), giving \( Den(\phi(t)) \).

Then the approximation, \( X_M(t) \), which consists of the low frequency part of the signal, \( S(t) \), is removed from the denoised signal. The result is the intermittent signal from the fuel assembly vibration.

\[ V(t) = Den(\phi(t)) - X_M(t) = T(t) \]  

The IR-index is calculated as the number of peaks in \( V(t) \). Each peak corresponds to the start of an intermittent fuel assembly vibration.

In order to use the wavelet method, a value of the eigenfrequency of the fuel assemblies has always to be given to perform the denoising to the correct level. Since the sampling frequency of measurement 1 only allows investigation up to frequencies of 6.25 Hz, the assumed eigenfrequency of the fuel assemblies is set to 5 Hz. Even though this value is far from realistic, it is the best that can be used when performing the wavelet analysis on this measurement.

As is commonly known when using wavelets, it is not easy to know which kind of wavelet to use for a certain analysing task. Using the Meyer wavelet in the MatLab Wavelet Toolbox
gave best agreement with the result from the spectral method. However, due to the low sampling frequency the judgment of which tubes that was most likely to impact is mainly based on the spectral analysis in measurement 1.

Fig. 11 shows the result from the spectral analysis of detectors at position 22, LPRM 22.2 and LPRM 22.4. LPRM stands for Local Power Range Monitor and is the notation of the neutron detectors used. LPRM 22.4 is the upper detector and LPRM 22.2 is the lower one. Clearly there is a broad peak in the APSD around the expected eigenfrequency of the detector tubes (1-2 Hz). Multiple peaks are also visible in the APSD. The coherence is high at the very same frequency and the characteristic distortion of the phase is present (non-linear and almost zero). Hence, this detector position was classified as one of the most likely to impact.

![Autospectra (APSD), coherence and phase for the detectors at LPRM position 22, measurement 1.](image)

After the analysis was performed, the plant personnel made visual inspections of some fuel assemblies, which were pointed out by the analysis, during the outage of the plant in August 2003. They inspected fuel assemblies around detector (LPRM) positions 1, 9, 10 and 22 out of which 1 had been classified by us as non-vibrating in the analysis and the three others were the ones that we judged most likely to impact. The result of the analysis and the inspection is presented in Table 1. As can be seen, LPRM 10 and 22 were pointed out by the visual inspection. They showed some wear marks on the corner of the fuel boxes. This is in good agreement with the prediction from the analysis.
4. Detector tube impacting

4.2.2 Measurement 2

The analysis of measurement 2 was performed in the same way as that of measurement 1. However, this time the sampling frequency, 64 Hz, was better suited for the wavelet method. The eigenfrequency of the fuel assemblies, was set to a more realistic value of 10 Hz in this case. In Fig. 12 a) the APSD, coherence and the phase of LPRM 16.2 and 16.4 is depicted and in b) $V(t)$ from equation (15) for LPRM 16.4 is shown. As can be seen, all the criteria for the spectral method are fulfilled and several spikes are visible in $V(t)$. Hence, this detector tube was pointed out as most likely to impact.

<table>
<thead>
<tr>
<th>Impacting status</th>
<th>LPRM by analysis</th>
<th>LPRM by inspection</th>
</tr>
</thead>
<tbody>
<tr>
<td>most likely</td>
<td>9, 10 and 22</td>
<td>10 and 22</td>
</tr>
<tr>
<td>probably</td>
<td>4, 8, 16 and 24</td>
<td></td>
</tr>
<tr>
<td>small chance</td>
<td>11, 12, 30 and 33</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 12.](image) a) Auto spectra (APSD), coherence and phase of detectors at LPRM position 16 from measurement 2. In b) the intermittent signal after the wavelet analysis of detector 16.4, measurement 2. The spikes indicating impacting are clearly visible.
Unfortunately, there was no time for visual inspections during the outage in 2004, since every second year there is a short outage. The results of the spectral and wavelet analysis are presented in Table 2. In the two groups of detectors with highest probability of impacting four out of six are pointed out by both methods, LPRM 16 24, 34 and 35. Thus, there is a good agreement between the two methods.

<table>
<thead>
<tr>
<th>Impacting status</th>
<th>By spectral analysis</th>
<th>By wavelet analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>most likely impacting</td>
<td>15, 16, 24 and 35</td>
<td>16, 23 and 24</td>
</tr>
<tr>
<td>probably impacting</td>
<td>12 and 34</td>
<td>4, 34 and 35</td>
</tr>
<tr>
<td>small chance of impacting</td>
<td>27 and 32</td>
<td>--------</td>
</tr>
</tbody>
</table>

### 4.3 Continuous wavelet transform

There is also a possibility to use the continuous wavelet transform in the analysis. In the case with detector tube impacting it could be of interest to calculate the so-called wavelet coherence, [28], between detectors in the same tube. Fig. 13 shows the wavelet coherence and the spectral coherence for two different detector positions.

![Fig. 13. Spectral, a), and continuous wavelet, b), coherence for detectors in position 1 and 16.](image-url)

Clearly, there is a much larger difference between the wavelet coherence in the frequency band between 10 to 20 Hz than it is in the spectral case. The presence of the large values in the wavelet coherence around 10-20 Hz is not fully understood. One guess is that it could be the
4. Detector tube impacting

fuel assembly vibrations that are seen. Detector positions pointed out by the wavelet IR-index also display large values in the frequency band between 10 and 20 Hz in the wavelet coherence. Even though the result from the continuous wavelet transform is not fully understood, this first result implies that further use may turn of to be of interest when investigating detector tube impacting.

4.4 Conclusions of the detector tube impacting

The spectral method is in good agreement with visual inspections. For the measurements with a sampling frequency that is suitable for the new wavelet based method, it is in agreement with the spectral one. One may wonder if there is need for a new method if a working one already exists. However, the advantage with this new wavelet method is that there is no need for comparison with a non-vibrating sample. It is an absolute method compared to the more relative spectral method, and its use requires no expertise, in contrast to the spectral based method, which cannot be performed e.g. by the control room personnel.

It is expected that a new measurement, with a higher sampling frequency that will be performed during the year of 2004/2005, and a visual inspection during the outage in late summer 2005 will confirm the agreement between the two methods and the real damage in the plant. The continuous wavelet coherence will be further investigated and hopefully better understood with the new measurement.

Up to now the wavelet method has been tested on signals from three Swedish plants, Barsebäck, Oskarshamn and Ringhals. It worked satisfactorily in all three cases, even though it had to be modified between its use in different plants.
5. Core-barrel vibrations

Core-barrel vibrations are, to a large extent, stationary processes, and have been successfully treated with spectral methods. The 2-D pendular vibrations, however, show sometimes intermittent properties (alternating isotropic - unilateral vibrations). Therefore, core-barrel vibrations will also be analysed with wavelet methods in the future. Accordingly, preparations are made by getting involved in the modelling of the in-core neutron noise induced by core-barrel vibrations. However, in this thesis, only classical, i.e. spectral analysis based, applications will be included. This analysis, still, has led to interesting new results in the understanding of certain experimental results.

Analyses of ex-core neutron noise of both beam-mode, ~8 Hz, and shell-mode, ~20 Hz, core-barrel vibrations have been used a long time, see Fig. 14, [3, 9-17]. It has also been noticed that the vibrations may lead to in-core noise which also can be used to analyse the vibrations. When analysing the shell-mode vibrations the use of in-core noise is especially important in Westinghouse reactors, since the ex-core detectors carry the same information, due to the 90° spacing. Therefore, the amplitude and the direction of the vibrations can not be determined at the same time. For this reason, in-core detectors have been included in the analysis of the shell-mode vibrations within a research project in cooperation with the Swedish NPP Ringhals [17]. However, the small number of in-core detectors hindered confirmation of the theory as well as use of the result. In order to have consistent interpretations it was necessary to assume that noise from in-core and ex-core detectors lying on the same azimuthal position have opposite phase. To confirm this out-of-phase behaviour, two different 1-dimensional analytical calculations of the noise induced shell-mode vibrations have been performed: an adiabatic approximation and a full space-frequency dependent solution. A comparison between the analytical results and the results from a numerical simulator, developed at the department [41], was also made. These calculations are described in detail in paper IV and V and a summary is given in this Section.

5.1 Description of the model

A 1-dimensional 2-group 2-region model is used with the boundary of the core set to \( b=161.25 \text{ cm} \) and the boundary of the reflector set to \( a=279.5 \text{ cm} \). The reactor parameters, e.g. cross-sections, are calculated from the core of the Pressurised Water Reactor (PWR).
Ringhals-4, using the in-core fuel management code SIMULATE-3 (homogenization from 3-D to 1-D). The choice of which reactor to use is arbitrary. One only needs some realistic parameters to visualize numerical results in the end. The fluxes are assumed to be symmetrical around the centre of the core.

Since a 1-D 2-group 2-region diffusion model is used, there are two diffusion equations in the core region, \( c \), and two diffusion equations in the reflector region, \( r \), for the static fluxes. One in each region for the fast flux, \( 1 \), and one for the thermal flux, \( 2 \).

\[
\begin{align*}
D_1^c \frac{d^2}{dx^2} \phi^c_1(x) - (\Sigma_{a,1}^c + \Sigma_R^c) \phi^c_1(x) + \frac{1}{k} \left( \nu \Sigma_{f,1}^c \phi^c_1(x) + \nu \Sigma_{f,2}^c \phi^c_2(x) \right) &= 0 \\
D_2^c \frac{d^2}{dx^2} \phi^c_2(x) - \Sigma_{a,2}^c \phi^c_2(x) + \Sigma_R^c \phi^c_1(x) &= 0
\end{align*}
\]

\text{core region (16)}

\[
\begin{align*}
D_1^r \frac{d^2}{dx^2} \phi^r_1(x) - (\Sigma_{a,1}^r + \Sigma_R^r) \phi^r_1(x) &= 0 \\
D_2^r \frac{d^2}{dx^2} \phi^r_2(x) - \Sigma_{a,2}^r \phi^r_2(x) + \Sigma_R^r \phi^r_1(x) &= 0
\end{align*}
\]

\text{reflector region (17)}

The cross-section notations are standard, i.e. \( a \) stands for absorption, \( f \) for fission and \( R \) for removal (scattering) from the fast group to the thermal. All the cross-sections and the diffusion coefficients are assumed to be constant within each region. The first step in calculating the phase behaviour of the noise, induced by shell-mode vibrations, is to solve the static equations (\( k \)-eigenvalue equations). That is easily done by using the symmetry, the interface (continuous fluxes and currents) and boundary (zero flux) conditions. Once a normalised and critical solution of the static fluxes is obtained the noise can be calculated. Criticality is achieved by adjusting the fission cross-sections.

5.2 The adiabatic approximation

In order to calculate the noise from shell-mode core-barrel vibrations, equations for a time-dependent system have to be used. One average group of delayed neutron precursors, \( C \), is assumed. The time-dependent diffusion equations now read

\[
\begin{align*}
\frac{1}{\nu_1} \frac{d\phi_1}{dt} &= \frac{\partial}{\partial x} D_1 \frac{\partial \phi_1}{\partial x} - (\Sigma_{a,1} + \Sigma_{s,1}) \phi_1 + (1 - \beta) (\nu \Sigma_{f,1} \phi_1 + \nu \Sigma_{f,2} \phi_2) + \lambda C \\
\frac{1}{\nu_2} \frac{d\phi_2}{dt} &= \frac{\partial}{\partial x} D_2 \frac{\partial \phi_2}{\partial x} - \Sigma_{a,2} \phi_2 + \Sigma_{s,1} \phi_1 \\
\frac{\partial C}{\partial t} &= -\lambda C + \beta (\nu \Sigma_{f,1} \phi_1 + \nu \Sigma_{f,2} \phi_2)
\end{align*}
\]

\text{(18)}

The arguments are left out, but everything except \( \beta, \nu \) and \( \lambda \) is both time- and space-dependent. The equations are valid in both the core and the reflector regions. The shell-mode vibration is modelled by letting the boundary between the core and reflector oscillate around the static positions, \( b \) and \(-b\), in a symmetrical way, \( b(t) = b + \epsilon(t) \) and \(-b(t) = -b - \epsilon(t)\).
The equations in (18) are solved by an adiabatic approach, i.e. splitting the fluxes into a time-dependent part, amplitude factor $P(t)$, and a space-time dependent part, shape function $\psi(x,t)$, as follows.

\[
\begin{align*}
\phi_1(x,t) &= P(t) \cdot \psi_1(x,t) & (P(0) = 1) \\
\phi_2(x,t) &= P(t) \cdot \psi_2(x,t) & (\psi_{1,2}(x,0) = \phi_{1,2}\text{static}(x))
\end{align*}
\]  

(19)

When using the adiabatic approximation a new normalization is needed. The shape function can be normalized by using the adjoint function. [42]. Using (19) in (18) and multiplying with the adjoint flux, subtracting the static equations and integrating over the reactor volume one ends up with the following equations after using the normalization condition.

\[
\begin{align*}
\frac{dP(t)}{dt} &= \frac{\rho(t) - \beta}{\Lambda(t)} P(t) + \lambda c(t) \\
\frac{dc(t)}{dt} &= \frac{\beta}{\Lambda(t)} P(t) - \lambda c(t)
\end{align*}
\]  

(20)

Linearising by splitting all time-dependent quantities into a static and a small deviation part, neglecting second order terms, $\delta P$ can be expressed in the frequency domain as:

\[
\delta P(\omega) = G_0(\omega) \cdot \delta \rho(\omega)
\]  

(21)

Here $G_0(\omega)$ is a transfer function, which is similar to the ordinary zero power reactor transfer function. Since the frequency of interest for the shell-mode vibrations, 20 Hz, is within the so-called plateau region the transfer function is constant and equal to:

\[
|G_0(\omega)| \approx \frac{1}{\beta}
\]  

(22)

Using this it is possible to express $\delta P$ in the time-domain and by inserting this in (19) and linearising, splitting into static and small deviation parts, neglecting second order terms. The expression of the space-time dependent neutron noise is:

\[
\begin{align*}
\delta \phi_1(x,t) &= \frac{\delta \rho(t)}{\beta} \cdot \phi_{1\text{static}}(x) + \delta \psi_{1\text{ad}}(x,t) \\
\delta \phi_2(x,t) &= \frac{\delta \rho(t)}{\beta} \cdot \phi_{2\text{static}}(x) + \delta \psi_{2\text{ad}}(x,t)
\end{align*}
\]  

(23)

Here the perturbed adiabatic shape function is used. The adiabatic approximation is valid if the perturbation is assumed to be small. Then the variation in time of the shape function is relatively small. It is then possible to calculate the perturbed adiabatic shape function at any time, as $\delta \psi_{\text{ad}}(x,t) = \psi_{\text{ad}}(x,t) - \phi_{\text{static}}(x)$, where $\psi_{\text{ad}}(x,t)$ is a static flux calculated from new k-eigenvalue equations, i.e. (16) and (17) with a larger core. $\delta \rho(t)$ is calculated, using the new $k$, as:

\[
\delta \rho(t) = 1 - \frac{1}{k(t)}
\]  

(24)
All this gives the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region reactor in the adiabatic approximation.

### 5.3 The full space-frequency dependent model

The other way of calculating the noise induced by the shell-mode vibration in a 1-D model is to make a full space solution in the frequency domain. First, the static cross-sections, in both the core and reflector region, are written as:

\[
\Sigma(x) = \{1 - H(x - b)\} \Sigma^c + H(x - b) \Sigma^r
\]  

(25)

Here \(H(x)\) is the unit step function. With a vibrating boundary \(b=b+\epsilon(t)\), the time-dependent cross-sections are written as:

\[
\Sigma(x, t) = \{1 - H(x - b - \epsilon(t))\} \Sigma^c + H(x - b - \epsilon(t)) \Sigma^r
\]  

(26)

Splitting the cross-sections into a static and a time-dependent part and making a Taylor expansion, assuming a small vibration, gives:

\[
\Sigma(x, t) = \Sigma(x) + \epsilon(t) \delta(x - b) \delta \Sigma, \quad \delta \Sigma = \Sigma^c - \Sigma^r
\]  

(27)

Using this together with a splitting into a static and a small deviation part of the fluxes and delayed neutron precursors in (18) and linearising by neglecting second order terms, equations in the frequency domain are obtained for the noise in the core and the reflector, which can be written in a condensed form as

\[
\hat{L}^c(x, \omega) \hat{\delta \phi}^c = \epsilon(\omega) \delta(x - b) \hat{S}(x) \hat{\phi}^{\text{static}}
\]

\[
\hat{L}^r(x, \omega) \hat{\delta \phi}^r = \epsilon(\omega) \delta(x - b) \hat{S}(x) \hat{\phi}^{\text{static}}
\]  

(28)

Here the vector fluxes in respective regions represent both the fast and thermal fluxes. The matrices \(S\) and \(L\) are given in paper V. On the right hand side in (28) it is not indicated if the static flux is taken from the core or the reflector. But since they are equal at the interface, \(x=b\), it does not matter which one is used. The neutron noise can be complex and the solution to (28) is given as:

\[
\begin{align*}
\hat{\delta \phi}^c(x, \omega) &= A_3 \begin{bmatrix} 1 \\ \tilde{C}_\kappa(\omega) \end{bmatrix} \sinh(\tilde{\kappa}_1(|x| - a)) + A_4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sinh(\tilde{\kappa}_2(|x| - a)) \\
\tilde{\delta \phi}^c(x, \omega) &= A_1 \begin{bmatrix} 1 \\ \tilde{C}_\mu(\omega) \end{bmatrix} \cos(\tilde{\mu}x) + A_2 \begin{bmatrix} 1 \\ \tilde{C}_\eta(\omega) \end{bmatrix} \cosh(\tilde{\eta}x) \\
\tilde{\delta \phi}^r(x, \omega) &= A_1 \begin{bmatrix} 1 \\ \tilde{C}_\mu(\omega) \end{bmatrix} \cos(\tilde{\mu}x) + A_2 \begin{bmatrix} 1 \\ \tilde{C}_\eta(\omega) \end{bmatrix} \cosh(\tilde{\eta}b)
\end{align*}
\]  

(29)

The boundary conditions of zero fluxes at the extrapolated boundaries \(x = \pm b\) have been used already in (29). The coefficients, \(\tilde{A}\), are determined from the following matrix equation:
The matrix $M$ consists of the interface conditions and is given in paper V. Now, the neutron noise from the shell-mode vibrations in a 1-D reactor is calculated in two different ways. The analytical solutions for the two cases are found in equations (23) and (29).

### 5.4 Comparison of numerical and analytical solutions

The thermal noise, denoted with subscript 2, is the one of interest, since the thermal neutrons are the ones detected by neutron detectors in the power plant. The phase from the second analytical solution is easily calculated from the complex neutron noise in (29). Since the noise is complex, it is possible to have all values between $\theta$ and $-\pi$. On the other hand, in the first case, with (23), the phase is calculated as either in-phase, $\theta$, or out-of-phase, $-\pi$, i.e. only two discrete values are possible. In Fig. 15 the thermal noise from both analytical solutions is shown. The most interesting feature is the large local out-of-phase behaviour close to the boundary, where the noise also has a large absolute value. The out-of-phase behaviour means that the thermal noise close to the interface has an opposite value as in the rest of the system, i.e. a decrease of the flux at the interface and an increase in the rest of the reactor. The global increase is due to the increase in reactivity, $\delta \rho > 0$. The decrease at the interface arises from the fact that fission material is added into the reflector region giving more absorption of thermal neutrons, and moderator material is removed, causing less slowing down of fast neutrons to thermal ones. Hence, the local decrease of the thermal flux in the calculations can be explained.

The thermal noise from the full space-frequency dependent analytical solution and the numerical simulator is shown in Fig. 16. The phase behaviour is very much the same in both cases, but there is some deviation in the absolute values. The difference in the absolute value is probably due to the fact that the numerical simulator uses nodes instead of continuous space. Hence, the perturbation at the boundary is spread out over a whole node rather than existing in a point (Dirac-delta function).

The conclusions of these calculations are that there is a large local component of the noise close to the interface between the reflector and the core and that it is indeed possible to have an out-of-phase behaviour between in-core, close to the reflector, and ex-core detectors. However, it is not clear if this out-of-phase behaviour is present in a 2-D model or not. Hence, the next step would be to expand the calculations into 2-D. If there is a large local out-of-phase behaviour close to the interface between the core and the reflector, it could be possible to measure it with detectors placed in the outermost fuel-assemblies. If this turns out to work it could be used as a diagnose tool for the shell-mode core-barrel vibrations.
Fig. 15. The absolute value, a), and the phase, b), of the thermal noise calculated with the two different analytical approaches.

Fig. 16. Absolute value, a), and phase, b), of the thermal noise from shell-mode core-barrel vibrations. The dotted line is calculated with the numerical simulator and the full line is the full space-frequency dependent analytical solution.
6. Concluding remarks

In this thesis three different problems were investigated, two of them within the area of classical reactor noise diagnostics, core-barrel vibrations and detector tube impacting. The relatively new mathematical tool, wavelet analysis, is used in two of the problems, detector tube impacting and two-phase flow identification. Especially the use of the continuous transform and the two-dimensional transform are new concepts in the research going on at the department. Both of the new wavelet methods seem promising for further development and application within the two projects and probably in other projects as well. Whenever there is non-stationary process, wavelet analysis could be of use. Hopefully, there will be several different new as well as old problems to tackle with the wavelets in the continuing work connected to the thesis.

As mentioned above in Sections 3 to 5, there is still some work to be done in each of the projects. As regards the core-barrel vibrations, the next step would be to extend the model into two dimensions and verify the results with measurements.

Concerning the detector tube impacting, there is already a new measurement under way. This new measurement will be performed during the ongoing fuel cycle of Ringhals-1 and hopefully there will be time for visual inspections during the next outage, which can verify the result. Moreover, there is need to explore the continuous wavelet transform to get a better understanding and knowledge of its features and possible applications. A physical interpretation of the results is of major importance for further use of the continuous transform.

The next step in the two-phase flow identification project would be to apply the energy feature extraction to noise-free neutron radiographic images. However, it is not clear whether it is possible to get such images or not.

The focus of using wavelets in the noise diagnostics will continue in both projects mentioned above and in other projects as well, e.g. estimation of MTC where a so-called wavelet spectrogram could be of use.

7. Acknowledgments

First of all I would like to thank my colleagues at the Department of Reactor Physics.

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8. References


8. References


9. Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
</tr>
<tr>
<td>APSD</td>
<td>Auto Power Spectral Density</td>
</tr>
<tr>
<td>BWR</td>
<td>Boiling Water Reactor</td>
</tr>
<tr>
<td>IR-index</td>
<td>Impact Rate index</td>
</tr>
<tr>
<td>LPRM</td>
<td>Local Power Range Monitor</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MTC</td>
<td>Moderator Temperature Coefficient</td>
</tr>
<tr>
<td>NPP</td>
<td>Nuclear Power Plant</td>
</tr>
<tr>
<td>PWR</td>
<td>Pressurised Water Reactor</td>
</tr>
<tr>
<td>STFT</td>
<td>Short Time Fourier Transform</td>
</tr>
</tbody>
</table>
| SKC          | Swedish Centre of Nuclear Technology
               | Svenskt Kärntekniskt Centrum |
| SKI          | Swedish Nuclear Power Inspectorate
               | Statens kärnkraftsinspektion |