Final Report on the Research Project
Ringhals Diagnostics and Monitoring
Stage 8

C. Demazière, C. Sunde, V. Arzhanov and I. Pázsit
Final Report on the Research Project
Ringhals Diagnostics and Monitoring
Stage 8

C. Demazière, C. Sunde, V. Arzhanov and I. Pázsit

Department of Reactor Physics, Chalmers University of Technology
SE-412 96 Göteborg, Sweden

Abstract

This report gives an account of the work performed by the Department of Reactor Physics in the frame of a research contract with Ringhals, Vattenfall AB, contract No. 531970-003. The contract constitutes Stage 8 of a long-term co-operative research work concerning diagnostics and monitoring of the BWR and PWR units. The work in Stage 8 has been performed between October 1st, 2002 and October 1st, 2003. In Stage 8 we have worked with four items as follows:

1. analysis of the LPRM measurements performed in Ringhals-1 on September 3-6, 2002 for the determination of possible unseated fuel assemblies;
2. analysis of the MTC measurements performed in Ringhals-2 on March 5-7, 2003;
3. calculation of the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region slab reactor;
4. analysis of the LPRM measurements performed in Ringhals-1 on September 3-6, 2002 for the investigation of detector string impacting.

This work was performed at the Department of Reactor Physics, Chalmers University of Technology by Imre Pázsit (project leader), Christophe Demazière, Carl Sunde, and Vasily Arzhanov. Contact person at Ringhals was Tell Andersson (project leader).
1. INTRODUCTION

The long term goal of the contract is to utilize the research potential of the Department of Reactor Physics at Chalmers in treating reactor physics problems related to the operation of the power plant. This is achieved in a co-operative research project, in the course of which the understanding of relevant problems increases at the department as well as methods are elaborated and implemented for their solution. The results obtained in the earlier stages have been reported in [1]-[8].

The work in Stage 8 has been performed between October 1st, 2002 and October 1st, 2003. It consisted of the following items, which are reported on in more detail in the following sections:

1. analysis of the LPRM measurements performed in Ringhals-1 on September 3-6, 2002 for the determination of possible unseated fuel assemblies;
2. analysis of the MTC measurements performed in Ringhals-2 on March 5-7, 2003;
3. calculation of the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region slab reactor
4. analysis of the LPRM measurements performed in Ringhals-1 on September 3-6, 2002 for the investigation of detector string impacting.

A further objective of the work is to gain experience with the important and characteristic features of power spectra and certain operational variables in the normal state, in order to be able to perform diagnostics of various phenomena.

A proposal for the continuation of the work in Stage 9 is also given at the end of this report.

2. ANALYSIS OF THE LPRM MEASUREMENTS PERFORMED IN RINGHALS-1 ON SEPTEMBER 3-6, 2002 FOR THE DETERMINATION OF POSSIBLE UNSEATED FUEL ASSEMBLIES

2.1. Introduction

On September 3-6, 2002, four measurements were carried out at the Ringhals-1 BWR. The first three measurements were performed on September 3, 2002 during start-up at a reduced core flow (about 4000 kg/s) and a reduced power level (about 70%), whereas the last measurement was performed on September 6, 2002 at full power and full core flow. Different signals were recorded during the measurements lasting a little bit more than 11 min each, at a sampling frequency of 12.5 Hz. The signals, which are used in this analysis, are the LPRM signals. All 36 LPRM detectors, each located on two different axial levels (on levels 2 and 4, 4 being the uppermost level), were monitored. The purpose of this analysis is to determine from these signals if there is any unseated fuel assembly in the core.

If there is one or several fuel assemblies in the core, then the LPRM signals will exhibit some characteristic features both in the time-domain and the frequency-domain. The two first parts of this report are thus devoted to the analysis of the LPRM signals in the time-domain, and the frequency domain respectively. The last part of this report presents the
results of a localization algorithm developed previously (see Refs. [9] and [10]), algorithm that is supposed to point out the location of unseated fuel assemblies if they exist.

2.2. Analysis of the measurements in the time domain

The first step of the analysis is to look at the time signals of all LPRMs. More precisely, it was demonstrated to be very useful to plot 2-D maps of the LPRMs belonging to the same axial level in the time-domain (see Ref. [9]). One thus obtains movies that are actually qualitatively very informative.

Prior to the plotting, it is essential to remove from each LPRM signal the components of the signal that do not correspond to the oscillation frequency. As will be seen in Section 2.3, the oscillation frequency was found to be equal to approximately 0.5 Hz. A classical infinite impulse response Butterworth filter was thus designed for that purpose (see Ref. [11]), the properties of which are summarised in the following Table 1. A non-causal, zero-phase filtering of the LPRM signals, which eliminates the non-linear phase distortion of an infinite impulse response filter, was then applied.

Table 1 Characteristics of the filter used in the analysis in the time-domain

<table>
<thead>
<tr>
<th>Pass-band frequency range (Hz)</th>
<th>Stop-band frequency range (Hz)</th>
<th>Attenuation in the pass-band frequency range (dB)</th>
<th>Attenuation in the stop-band frequency range (dB)</th>
<th>Order of the filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4 - 0.6</td>
<td>0.0 - 0.35/0.65 - 6.25</td>
<td>1</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

It was noticed that all the signals clearly showed a fundamental or global (in-phase) oscillation due to the time delay of the void/pressure response to any reactivity perturbation. In order to better visualize the effects of possible unseated fuel elements on the LPRM signals, it was then decided to remove the in-phase behaviour or point-kinetic response of the reactor from all the signals as explained in the following. The flux was first factorised into an amplitude function $P(t)$ and a shape function $\psi(r, t)$ as follows

$$\phi(r, t) = P(t)\psi(r, t)$$  \hspace{1cm} (1)

where

$$\frac{\partial}{\partial t}\int_0^r \phi_0(r)\psi(r, t)dr = 0$$  \hspace{1cm} (2)

and

$$\phi_0(r) = \psi(r, t = 0)$$  \hspace{1cm} (3)

From Eqs. (2) and (3), one can see that the fluctuation of the shape function and the static flux are orthogonal:

$$\int \phi_0(r)\delta\psi(r, t)dr = 0$$ \hspace{1cm} (4)

The flux noise in the frequency domain can thus be approximated by (second-order terms
neglected): 

\[ \delta \phi(r, t) = \delta \phi_{pk}(r, t) + \delta \psi(r, t), \]  

where 

\[ \delta \phi_{pk}(r, t) = \phi_0(r) \delta P(t) \]  

is the point-kinetic component of the flux noise. Due to the orthogonality between the fluctuation of the shape function, i.e. \( \delta \psi(r, t) \), and the static flux \( \phi_0(r) \) [see Eq. (4)], the point-kinetic component of the flux noise can be determined from 

\[ \delta P(t) = \frac{\delta \phi_{pk}(r, t)}{\phi_0(r)} = \frac{\int \delta \phi(r, t) \phi_0(r) dr}{\int \phi_0^2(r) dr} \]  

If one has access to many radial in-core neutron detectors, Eq. (7) can be approximated by replacing the integrals by a sum over a number \( N \) of detectors, as follows: 

\[ \delta P(t) = \frac{\delta \phi_{pk}(r, t)}{\phi_0(r)} \approx \frac{\sum_{i=1}^{N} \delta \phi(r_i, t) \phi_0(r_i)}{\sum_{i=1}^{N} \phi_0^2(r_i)} \]  

The remaining response is consequently simply given as: 

\[ \delta \psi(r, t) = \delta \phi(r, t) - \delta \phi_{pk}(r, t) = \delta \phi(r, t) - \phi_0(r) \delta P(t) \]  

which actually corresponds to the fluctuations of the shape function \( \psi(r, t) \) in linear theory.

Movies were then created displaying both the total flux noise \( \delta \phi(r, t) \), the point-kinetic component or global oscillation \( \delta \phi_{pk}(r, t) \), and the remaining part or fluctuations of the shape function \( \delta \psi(r, t) \). It was seen in the measurements \( a, b, \) and \( c \) that the fluctuations of the shape function clearly show a first azimuthal mode or regional (out-of-phase) behaviour, due to the time delay of the void/pressure response to any reactivity perturbation via the recirculation loop dynamics, as can be noticed in Figs. 1 - 3, which are snapshots of the movies mentioned above. Such a behaviour is not as clearly visible in measurement \( d \), probably due to the fact that the core is operating at nominal core flow and power, whereas the three first measurements were carried out at reduced power and core flow. Regarding the regional oscillation, a symmetry line is also plotted, calculated as the line perpendicular to the segment joining the average positions of the positive and negative flux noise at a given time, respectively. It was noticed that this symmetry line was actually rotating. In formula, the average positions are given as:

\[ r_+ = \frac{\sum_{k, \delta \psi(r_k, t) > 0} \delta \psi(r_k, t) r_k}{\sum_{k, \delta \psi(r_k, t) > 0} \delta \psi(r_k, t)} \]
and

\[
 r_+ = \frac{\sum_{k, \delta \psi(r_k, t) > 0} \delta \psi(r_k, t) r_k}{\sum_{k, \delta \psi(r_k, t) < 0} \delta \psi(r_k, t)}
\]

for the positive and negative lobes, respectively. The intersection between the symmetry line and the segment joining the average positions of the positive and negative lobes is a weighted-average of the two previous average positions, i.e.

\[
 r_\perp = \frac{r_+ \sum_{k, \delta \psi(r_k, t) > 0} \delta \psi(r_k, t) + r_- \sum_{k, \delta \psi(r_k, t) < 0} |\delta \psi(r_k, t)|}{\sum_k |\delta \psi(r_k, t)|}
\]

No channel instability or Density Wave Oscillation (DWO) or local oscillation due to a purely thermal-hydraulic feedback effect can be noticed, since there is no detector noise that seems to drive the response of the other detectors. If a fuel element was unseated, some of the coolant flow would bypass the fuel element and this would render the channel thermal-hydraulically unstable. There is absolutely no evidence of such an effect in the movies. For comparison purposes, one can refer to the so-called Forsmark-1 channel instability event encountered during the start-up tests of the fuel cycle 16 in 1997, where at least one fuel assembly was unseated (this unseated fuel assembly was discovered during the fuel outage that followed the fuel cycle 16). A snapshot of a movie representing the neutron noise measured by the LPRMs at a given axial level is depicted in Fig. 4 for the Forsmark-1 case. It can be seen in this Figure that the amplitude of the noise is maximum at a given detector position, and decreases rapidly away from that detector, therefore suggesting the spatial attenuation of the local noise source. This case does not correspond at all to the Ringhals-1 case.
Fig. 1. Snapshot of the movie displaying the response of the LPRMs in the time-domain, for the measurement \( a \).
Fig. 2. Snapshot of the movie displaying the response of the LPRMs in the time-domain, for the measurement $b$. 
Fig. 3. Snapshot of the movie displaying the response of the LPRMs in the time-domain, for the measurement c.
Fig. 4. Snapshot of the movie displaying the response of the LPRMs in the time-domain, for the Forsmark-1 channel instability event.
Another way of finding out unseated fuel assemblies is to look at the standard deviation of all LPRMs plotted on 2-D radial maps. If there was an unseated fuel assembly, the level of noise of the nearest LPRMs would be definitely higher. As can be seen in Figs. 5 - 8, no LPRM shows a standard deviation much larger than the other ones, thus indicating that the probability of having an unseated fuel assembly is very small.
Fig. 5. Standard deviation of the LPRM signals for the measurement a.

Fig. 6. Standard deviation of the LPRM signals for the measurement b.
Fig. 7. Standard deviation of the LPRM signals for the measurement \( c \).

Fig. 8. Standard deviation of the LPRM signals for the measurement \( d \).
2.3. Analysis of the measurements in the frequency domain

Another way of finding unseated fuel elements is to look at the spectral features of the signals. All the LPRM signals were thus analysed in the frequency domain. In Figs. 9 - 12, the LPRM string 20 is taken as an illustration of the general characteristics of all LPRM strings. As can be seen in these Figures, there is a peak in the Auto-Power Spectral Density (APSD) of any LPRM, corresponding to the 0.5 Hz oscillation that was visualized in the time-domain in Section 2.2, for all measurements except the measurement d. The reason of the difference between the three first and the last measurements lies with the operating conditions of the different measurements. The first three ones were carried out during start-up, i.e. at reduced power and core flow, conditions favourable to instabilities, whereas the last measurement was performed at full power and core flow, where in principle instability conditions should not occur. Regarding the other characteristics of the signals, one notices a normal (gaussian) distribution of the noise, a high coherence between detectors belonging to the same strings for frequencies lower than 0.5 Hz, a linear phase behaviour of the Cross-Power Spectral Density (CPSD) between these detectors due to the transport of the perturbation upwards with the flow, i.e. due to the time delay between the two detectors it takes to measure the same perturbation. During the analysis of the signals, it was also noticed that the LPRM 34, level 4 was faulty (no spectrum signature, very low level of noise, no correlation with the LPRM 34, level 2). Some other characteristics were also found for some detector strings, but at frequencies much higher than 0.5 Hz. These characteristics are not related to the instability studied in this report. Consequently, they are not touched upon here.
Fig. 9. Characteristics of the neutron noise measured by the LPRM string 20 for the measurement $a$. 
Fig. 10. Characteristics of the neutron noise measured by the LPRM string 20 for the measurement b.
Fig. 11. Characteristics of the neutron noise measured by the LPRM string 20 for the measurement \( c \).
Fig. 12. Characteristics of the neutron noise measured by the LPRM string 20 for the measurement $d$. 
A further analysis of the spectral properties of the LPRMs in order to find out if the core contains unseated fuel elements is to plot 2-D maps of the phase of the CPSD between detectors belonging to the same axial level. Since a global oscillation was clearly demonstrated in Section 2.2, it is essential to remove this in-phase oscillation prior to the analysis of the spatial structure of the phase of the CPSD. If there was an unseated fuel assembly, the phase would be roughly homogeneous throughout the core, or more precisely slightly decreasing away from the unseated fuel assembly. Such an example is presented in Section 13, which is the result of simulations where an unseated fuel element was modelled by an “absorber of variable strength” type of noise source. As can be seen in Figs. 15 - 17, the spatial structure of the CPSD angle is very much different from the one presented in Fig. 13, therefore again suggesting that there no unseated fuel element in the core. As a matter of fact, an out-of-phase behaviour corresponding to the regional oscillation can clearly be noticed.

**Fig. 13.** Typical phase of the flux noise in case of an unseated fuel element (the unseated fuel element is located where the phase is at its maximum).
Fig. 14. CPSD angle between the LPRMs for the measurement a.

Fig. 15. CPSD angle between the LPRMs for the measurement b.
Phase shift between LPRM 20 and the other LPRMs, level 2

Phase shift between LPRM 20 and the other LPRMs, level 4

Fig. 16. CPSD angle between the LPRMs for the measurement c.

Fig. 17. CPSD angle between the LPRMs for the measurement d.
Another proof of the absence of unseated fuel assemblies is actually the relatively weak space-dependence of the Decay Ratio (DR). As can be seen in Figs. 18 and 19, the DR calculated by Ringhals for the measurements \( b \) and \( c \) is space-dependent, but relatively homogeneous radially. This is in clear contrast with the Forsmark-1 channel instability event mentioned previously. In this event, the DR was strongly space-dependent, with clearly two distinct regions in the core: one region with a DR of about 0.5, and one region with a DR of about 0.9. Such a space-dependence is recalled in Fig. 20. Furthermore, a recent theoretical investigation (see Ref. [13]) demonstrated that only the presence of at least one unseated fuel assembly, i.e. a local instability, coexisting with another kind of oscillation (either a global one, or a regional one, or another local one) could explain a radially strongly space-dependent DR. The results of simulations carried out with a local noise source coexisting with a global oscillation are depicted in Fig. 21, whereas the results corresponding to two coexisting local noise sources are depicted in Fig. 22. As can be seen in these two Figures, this is only the fast spatial decay of the amplitude of the local oscillations that allow explaining the strong space-dependence of the DR throughout the core. The Ringhals-1 measurements do not resemble these different cases at all, therefore suggesting that there is no unseated fuel assembly. Some further calculations carried out with a global oscillation coexisting with a regional oscillation, case which seems to be closer to the Ringhals-1 measurements, actually revealed that the DR is weakly space-dependence, as was precisely noticed in Figs. 19 and 20. The results of such calculations are depicted in Fig. 23.
Fig. 18. Radial space-dependence of the DR in the Ringhals-1 case for the measurement $b$.

Fig. 19. Radial space-dependence of the DR in the Ringhals-1 case for the measurement $c$. 
2.4. Search for unseated fuel assemblies

The Department of Reactor Physics, Chalmers University of Technology developed a so-called localisation algorithm (see Refs. [9] and [10]), i.e. a procedure that allows locating a local noise source from the LPRM readings. Although there is clear indication from the foregoing that it is unlikely that the Ringhals-1 core contains any unseated fuel assembly, it was found interesting to apply the localisation algorithm to the Ringhals-1 case.

The localisation procedure gives the location of the noise source (if any) existing in the core, not its strength. Actually the localisation is achieved by searching of the minimum of the following function

\[ \Delta(r) = \sum_{A, B, C, D} \Delta^2_{A, B, C, D}(r) \]

with

\[ \Delta_{A, B, C, D}(r) = \frac{CPSD_{A, B}(\omega)}{CPSD_{C, D}(\omega)} \frac{G_{\Sigma_{rem} \rightarrow 2}(r, \omega) \times G_{\delta \Sigma_{rem} \rightarrow 2}^*(r, \omega)}{G_{\Sigma_{rem} \rightarrow 2}(r, \omega) \times G_{\delta \Sigma_{rem} \rightarrow 2}^*(r, \omega)} \]

where \((A, B, C, D)\) represents a quadruplet of neutron detectors. \(G_{\delta \Sigma_{rem} \rightarrow i}(r, r', \omega)\) is the 2-

Fig. 20. Radial space-dependence of the DR in the Forsmark-1 channel instability event (derived from Ref. [12]).
Fig. 21. Simulated radial space-dependence of the DR in case of a local noise source and a global noise source (the white square represents the location of the local noise source).
Fig. 22. Simulated radial space-dependence of the DR in case of two local noise sources (the white squares represent the location of the local noise sources).
Fig. 23. Simulated radial space-dependence of the DR in case of a global noise source and a regional noise source.
D 2-group discretised Green’s function, the index \( i = 1, 2 \) representing the fast and thermal groups, respectively. More specifically, these transfer functions give the flux noise \( \delta \phi_i \) in \( r \) and at a frequency \( f = \omega / 2\pi \) induced by a unit removal cross-section noise source \( \delta \Sigma_{rem} = 1 \) located at \( r' \) at the same frequency. This transfer function was estimated by using a neutron noise simulator developed at the Department of Reactor Physics, Chalmers University of Technology (see Ref. [14]), and was calculated for the Ringhals-1 core layout, but with a set of homogeneous cross-sections/material constants representative of a General Electric BWR/6 [equilibrium core at beginning of cycle conditions (see Ref. [15])]. Simply speaking, the \( \Delta(r) \) function compares the detector readings, or more precisely their CPSDs, to the corresponding calculated flux noise induced by a noise source located in \( r \). The ratio between the CPSDs is taken so that the noise source strength, which is unknown, can be eliminated. The function \( \Delta(r) \) has to be calculated for every possible location \( r \) of the noise source in the core. Therefore, the minimum of this function should correspond to the location of the actual noise source in the core. It has to be emphasized that the localisation procedure implicitly assumes the presence of a local noise source in the core, i.e. the algorithm is not able to predict if there is an unseated fuel assembly or not. Such a localisation algorithm was tested on simulated data and was demonstrated to give the correct location of the noise source regardless of how many detectors are used, of the actual location of the noise source in the core, and of the contamination of the detector readings by extraneous noise as long as there is one single noise source present in the core. The localisation algorithm was designed for locating one single noise source, and therefore if two or more noise sources are present at the same time in the core, the location of the noise source returned by the algorithm is biased. Nevertheless, simulations carried out with two noise sources revealed that choosing a set of detectors positioned close to one of the noise sources allows detecting successfully the corresponding one, as long as the two noise sources are not close enough. An attempt to locate the noise source in the Forsmark-1 channel instability event was carried out in the past (see Refs. [9] and [10]). The corresponding \( \Delta(r) \) function is recalled in Fig. 24, where the detectors are positioned via crosses (‘X’), the white ones indicating the detectors used in the localisation, the black ones the detectors not used. The position pointed out by the localisation algorithm is very close to a fuel assembly that was found to be unseated during the following core outage.

Nevertheless, choosing a different set of detectors gives results which are sometimes different, i.e. the noise source is not always located at a position close to the unseated fuel element. This suggests that there are probably two (or maybe even more) noise sources located inside the core. This can be seen in Fig. 25, where the \( \Delta(r) \) function obtained with all the available detectors is plotted. As pointed out previously, limiting the number of detectors to a region where a noise source is suspected to be located, as was done in Fig. 24, allows successfully locating this specific noise source, as long as the other noise sources are not in the same vicinity. This is why the region around the unseated fuel element was pointed out by the localisation algorithm. Taking more detectors into account than the one used in Fig. 24 is equivalent to take the effect of several other possible noise sources into account, whereas the algorithm has been designed for a single noise source. It is also worth mentioning that only the region pointed out by the localisation algorithm was visually inspected during the core outage, i.e. other unseated fuel elements might have remained undetected. It is finally important to point out that the \( \Delta(r) \) function is extremely flat throughout the core, except at the location of the noise sources (minima) and at the location of the detector strings (maxima). This behaviour should be considered as typical when the core contains unseated fuel assemblies.
Fig. 24. Result of the localisation algorithm in the Forsmark-1 case (local instability event) when using a reduced set of detectors; the unseated fuel element is marked with a square, and the noise source identified by the localisation algorithm with a circle.
Fig. 25. Result of the localisation algorithm in the Forsmark-1 case (local instability event) when using all the available detectors. Two noise sources (represented by the dark-blue regions) seem to be present in the core.
Based on the previous experience with the Forsmark-1 channel instability event, the following methodology was applied to the Ringhals-1 case. First, each measurement set was used as input data to the localisation procedure, for the levels 2 and 4 respectively, by using many LPRMs distributed evenly throughout the core. As pointed out in the Forsmark-1 case, these calculations should indicate some tendencies, but will not necessarily point out the exact location of the noise source, if there are for instance several noise sources. The results of these preliminary calculations are presented in Figs. 26 - 33, where the detectors are positioned via crosses (‘X’), the white ones indicating the detectors used in the localisation, the black ones the detectors not used. It can be noticed that the localisation procedure tends to locate the noise outside the core (the noise source pointed out by the algorithm is designed to be found in the fuel region, but the location pointed out by the search algorithm is always close to the reflector nodes). In two occurrences, the noise source was found in the middle of the core approximately. It is essential to emphasize that in all these cases, the shape of the $\Delta(r)$ function is very much different from the Forsmark-1 channel instability event depicted in Fig. 25. The $\Delta(r)$ function does not show any clear minimum, and is smoothly varying throughout the core. This suggests that the probability of unseated fuel assemblies is very small in the Ringhals-1 case.
Fig. 26. Result of the localisation algorithm in the Ringhals-1 case (measurement a, level 2) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 27. Result of the localisation algorithm in the Ringhals-1 case (measurement α, level 4) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 28. Result of the localisation algorithm in the Ringhals-1 case (measurement $b$, level 2) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 29. Result of the localisation algorithm in the Ringhals-1 case (measurement b, level 4) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 30. Result of the localisation algorithm in the Ringhals-1 case (measurement c, level 2) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 31. Result of the localisation algorithm in the Ringhals-1 case (measurement c, level 4) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 32. Result of the localisation algorithm in the Ringhals-1 case (measurement $d$, level 2) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
Fig. 33. Result of the localisation algorithm in the Ringhals-1 case (measurement $d$, level 4) when using evenly distributed detectors; the noise source identified by the localisation algorithm is marked with a square.
It was nevertheless decided to perform more detailed calculations, i.e. by dividing the core into nine regions, applying the localisation algorithm to each of these regions using three to four detectors for all the measurements and all the available axial levels. The nine different regions are represented in Fig. 34. If a noise source was found in a given region, then the algorithm was again applied to that specific region by using the closest detectors to the noise source. If the noise source was then found to be lying outside that region, the noise source was considered to be unrealistic. Otherwise, the noise source might actually correspond to an unseated fuel assembly, although such a possibility is very weak. The results of all the calculations are summarized in Table 2 below. It seems from this Table that if there are unseated fuel assemblies, they might be located in either regions III, IV, VIII, or IX. From the noise sources pointed out by the localisation algorithm in each region for the different measurements, it is then possible to locate the average position of the noise sources in each region. Such an estimation is presented in Fig. 35. Therefore, if it is decided to visually inspect the Ringhals-1 core for finding out unseated fuel assemblies (even if the probability of having unseated fuel assemblies seems to be very small), then it should be around the regions depicted in Fig. 35.

Table 2 Summary of the location of the noise sources found when using reduced sets of detectors (the crosses indicate that a “reasonable” noise source might exist in that specific region).

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Region I</th>
<th>Region II</th>
<th>Region III</th>
<th>Region IV</th>
<th>Region V</th>
<th>Region VI</th>
<th>Region VII</th>
<th>Region VIII</th>
<th>Region IX</th>
</tr>
</thead>
<tbody>
<tr>
<td>a, level 2</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>a, level 4</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b, level 2</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>b, level 4</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c, level 2</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>c, level 4</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d, level 2</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>d, level 4</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 34. Location of the different regions for the localisation algorithm, when using reduced sets of detectors.
Fig. 35. Regions of interest for visual inspection in case of the assumption of possible unseated fuel assemblies (the crosses represent the detector strings location, and the black squares the possible unseated fuel assemblies).
2.5. Conclusion

All the analyses reported in this study seem to indicate that there was no unseated fuel assembly in Ringhals-1 when the measurements were carried out in September 2002. These analyses were performed both in the time-domain, and in the frequency domain. In the frequency domain, a localisation algorithm was even used in order to determine the location of possible unseated fuel assemblies. No conclusive location was ever found. Nevertheless, since the localisation procedure always finds a minimum of the $\Delta(r)$ function that should correspond to a local noise source, even if there is no unseated fuel assembly, all the measurements were analysed for both axial levels. By dividing the core into several regions, some common features between all measurements were discovered. More precisely, four regions of the core are worth being looked at if a visual inspection of the assemblies is considered. It is very unlikely that any unseated fuel assembly would be found in these regions, but if there are unseated fuel elements, then they have to be located close to the fuel assemblies pointed out in Fig. 35.

3. ANALYSIS OF THE MTC MEASUREMENTS PERFORMED IN RINGHALS-2 ON MARCH 5-7, 2003

3.1. Introduction

In Stage 7 (see [8]), a new method for determining the at-power Moderator Temperature Coefficient in PWRs based on noise analysis was developed. This technique was also tested via a noise measurement performed in 2002 at Ringhals-2. This method relies on a new MTC noise estimator which can be written as:

$$H_{1,\text{new}}^{\text{biased}}(r, \omega) = \beta_{\text{eff}} \frac{CPSD_{\delta \phi/\phi_0, \delta T_m^\text{ave}(r, \omega)}}{APSD_{\delta T_m^\text{ave}(\omega)}}$$  \hspace{1cm} (15)

This MTC evaluation should be carried out at frequencies lying in the band 0.1 - 1.0 Hz, while the flux noise and the moderator temperature noise should be measured on the same axial plane of the reactor. Practically, the flux noise is measured by in-core neutron detectors, whereas the radial distribution of the moderator temperature noise is measured by the Ringhals-2 Gamma-Thermometers (GTs), which in the frequency band 0.1 - 1.0 Hz are actually working as ordinary thermocouples. Several weighting functions could be used for calculating the radially-averaged moderator temperature noise:

$$\delta T_m^\text{ave}(\omega) = \frac{\int \delta T_m(r, \omega) w(r) dr}{\int w(r) dr}$$  \hspace{1cm} (16)

It was suggested that using the square of the static value of the GTs was actually giving the best results, and was also a practical solution, since the estimation of the weighting function would otherwise require core calculations. This new MTC noise estimator is nevertheless biased due to the fact that the point-kinetic component of the flux noise, which should be used in practise to get the correct MTC value, cannot be measured via a single in-core neutron detector. This bias was nevertheless demonstrated to be very small. The MTC was therefore correctly estimated in March 2002 in Ringhals-2 by using one in-core neutron...
detector and 12 Gamma-Thermometers (GTs) located on the same radial plane of the reactor.

Although this new technique was very encouraging for estimating the at-power MTC without any disturbance of the reactor operation, a few problems were pointed out. First of all, it was noticed that the results were very much dependent on the number of points used in the Fast Fourier Transform (FFT) in the processing of the signals. This behaviour is typical of bad signal statistics, i.e. measurements that are too short and/or inaccurate measurement of the noise. In the 2002 measurement, the measurement were about 20 min long at a sampling frequency of 8 Hz. In principle, the number of blocks used for the FFT calculations should be larger than 100 [16]. For the 2002 measurement, this condition was obviously not fulfilled with 256 FFT points, which is a standard number for signal processing. The fact that the measurement is relatively short, i.e. 20 min typically, is due to the use of in-core neutron detectors for measuring the flux noise. In PWRs, these in-core neutron detectors are fission chambers, which cannot be used for a too long time, since their efficiency would otherwise change appreciably due to the depletion of the fissile material in the detectors. It was therefore suggested to Ringhals to perform noise measurement using ex-core neutron detectors (B-10 ionization chambers) instead of in-core neutron detectors. Another possibility would be to use several in-core neutron detectors located near to each other and to insert them alternatively, 20 min for each of them. It was also recommended to Ringhals to remove the mean value of the measured signals before the analog-to-digital conversion of the signals, thing which had not been done in the 2002 measurement. This would increase significantly the accuracy of the measured noise. Secondly, although the reactor should behave in a manner close to point-kinetics, the new MTC noise estimator is still biased due to the fact that only the total flux noise can be measured by one single in-core neutron detector. As will be illustrated later on in this Section, the point-kinetic component of the flux noise can be approximately reconstructed when the flux noise is measured simultaneously at several locations of the core. It was thus recommended to Ringhals to use several in-core neutron detectors simultaneously. Finally, another drawback of the new MTC noise estimator is the fact that the radial distribution of the moderator temperature noise should be measured inside the core. This was done in Ringhals-2 via the use of GTs. It is nevertheless known that western-type PWRs do not ordinarily have in-core thermocouples, but only core-exit thermocouples. It was thus decided to investigate the possibility of using the radial average of the core-exit thermocouples instead of the radial average of the GTs. According to the results of the previous investigation reported in Stage 7 [8], using the radial average of the core-exit thermocouples would not allow determining the actual MTC, since the core-exit moderator temperature noise is fundamentally different from the in-core moderator temperature noise (effect due to the coolant mixing above the fuel assemblies). Nevertheless, it was suggested to Ringhals to perform also a measurement with core-exit thermocouples in order to check this assumption.

If one tries to summarize, a new measurement campaign was suggested to Ringhals, with the following goals:

- improvement of the measuring technique tested in 2002;
- investigation of the possibility of using ex-core neutron detectors;
- investigation of the possibility of using core-exit thermocouples;
- investigation of the possibility of using several in-core neutron detectors inserted successively in the core.
Such a measurement campaign was carried out at the Ringhals-2 PWR between March, 5-7, 2003 [17]. Three different measurements were actually performed, at a sampling frequency of 16 Hz. Each of these measurement is reported in detail in the following paragraphs, from which conclusions are drawn. Although no core data were provided to the Department of Reactor Physics, Chalmers University of Technology, the effective fraction of delayed neutrons was assumed to be roughly identical to the one calculated at the occasion of the 2002 measurement, i.e. 536 pcm, whereas the MTC was assumed to be about -50 pcm/°C. The signal processing was performed with 512 FFT points, since the length of each of the measurements did not allow using more than 512 FFT points (which would have led to an insufficient number of blocks).

3.2. Analysis of the measurement number 1

The measurement set-up for the measurement number 1 is depicted in Fig. 36 and is as follows:
• all 29 gamma-thermometers (12 on the lower plane − plane 7 −, 10 on the upper plane − plane 3 −, and 9 in the fuel assembly J10) were monitored;
• one ex-core neutron detector monitoring the lower half of the core;
• the measurement length was equal to approximately 1 hour.

The radial average temperature noise on the 3rd and the 7th planes from the top of the core active height was then calculated. Prior to this calculation, it could be interesting to determine a calibration factor for the “old” MTC noise estimator, the MTC noise estimator

Fig. 36. Radial location of the detectors on the left hand-side, and axial location on the right hand-side, with the fuel assembly J10 axially fully instrumented with gamma-thermometers, for the measurement number 1.
that is based on the use of the temperature noise measured in one signal radial location of the core:

\[ H_{1, \text{old}}^{\text{biased}}(r, \omega) = \frac{1}{G_0(\omega)} \frac{\text{CPSD}_{\delta\phi/\delta T, m}(r, \omega)}{\text{APSD}_{\delta T, m}(r, \omega)} \]  

(17)

As mentioned in Stage 7 [8], this “old” MTC noise estimator is systematically biased low since the moderator temperature noise is radially loosely coupled in the core. This calibration factor was already derived in Stage 7, and is given by:

\[ \frac{\text{APSD}_{\delta T, m}(r, \omega)}{\sqrt{\text{APSD}_{\delta T, m}(r, \omega)}} = \frac{\text{APSD}_{\delta T, m}^{\text{ave}}(\omega)}{\delta T_m(r, \omega)} = \frac{H_{1, \text{old}}^{\text{biased}}(r, \omega)}{H_{1, \text{new}}^{\text{biased}}(r, \omega)} \]  

(18)

The estimation of such a calibration factor is depicted in Figs. 37 and 38, on the 3rd and 7th axial plane respectively, assuming that the local measurement of the temperature noise is carried out in the fuel assembly J10. As can be seen in these two Figures, measuring the moderator temperature noise locally would systematically underestimate the MTC by a factor of 2 to 3.

The MTC can finally be estimated by using the new MTC noise estimator, as depicted in Fig. 39. For that purpose, the radial average temperature noise was estimated from the GT signals. Another difference with the measurement reported in Stage 7 is the fact that an ex-core neutron detector located in the neighbourhood of the 7th plane from the top of the core active height was used. In order to get the MTC magnitude, one should consider in the
frequency band 0.1 - 1.0 Hz both the coherence and the phase between the average temperature noise and the neutron noise. The coherence should obviously be large enough, and the phase should be close to ±180 deg since the MTC is a negative reactivity coefficient in the present case. These considerations allow pointing out the frequencies at which the noise estimator should give a correct MTC estimation. Averaging over all these frequencies, one finds an MTC magnitude in the vicinity of 50 pcm/°C.

Finally, since the 9 GTs of the detector string J10 were measured, one can estimate the axial dependence of the temperature noise in the fuel channel J10. Such an estimation is presented in Fig. 40. The higher level of noise at the bottom of the core is due to a peak in the APSD of the GT at that level of the core at a frequency of 1.5 -1.6 Hz. The reason of this peak in the APSD is not clearly understood. This perturbation is nevertheless axially damped since its period is shorter than the axial propagation time, thus giving a rather flat distribution of the moderator temperature noise above that level. This suggests again that the temperature noise is created outside the core, probably at the core inlet, thus explaining why the axial dependence of the moderator temperature noise can be disregarded when performing the MTC evaluation.
Fig. 39. MTC estimation by using the radial average of the GT signals on the 7th plane from the top of the core active height and by using an ex-core neutron detector.
Fig. 40. Axial dependence of the temperature noise in the fuel channel J10.
3.3. Analysis of the measurement number 2

The measurement set-up for the measurement number 2 is depicted in Fig. 41 and is as follows:

- all 12 gamma-thermometers located on the lower plane (plane 7) and all core-exit thermocouples were monitored;
- 4 in-core neutron detectors located on the 7th plane from the top of the core active height were used;
- the measurement length was equal to approximately 20 min.

As before, it is possible to estimate a calibration factor so that the radial average temperature noise in the core can be substituted by the core-exit radial average temperature noise. This calibration factor can be calculated as:

$$\frac{\text{APSD}}{\text{APSD}} \frac{\delta T_{m, \text{ave}, \text{in-core}}(\omega)}{\delta T_{m, \text{ave}, \text{core-exit}}(\omega)} = \frac{\delta T_{m, \text{ave}, \text{in-core}}(\omega)}{\delta T_{m, \text{ave}, \text{core-exit}}(\omega)} = \frac{H_{1, \text{new}}^{\text{biased, core-exit}}(r, \omega)}{H_{1, \text{new}}^{\text{biased, in-core}}(r, \omega)}$$  \hspace{1cm} (19)

If it was possible to use the core-exit radial average temperature noise in the new MTC estimator, then this calibration factor would be close to unity. As can be seen in Fig. 42, the value of this calibration factor is about 10, i.e. using the core-exit thermocouples instead of the GTs would underestimate the MTC by a factor of roughly 10. It therefore means that using the core-exit thermocouples for determining the MTC by noise analysis is impossible without any calibration. This reason of this bias is due to the fact that the moderator temperature noise is different at the core-exit than inside the core. The coolant mixing above the fuel assemblies is probably responsible for this effect.
Before evaluating the MTC by using the radial average of all core-exit thermocouples, one could check that using the in-core thermocouples, i.e. the GTs, provides a correct estimation of the MTC. For that purpose, the MTC was first evaluated by using the radial average temperature noise measured by the GTs for all possible in-core neutron detectors respectively. Such estimations are given in Figs. 43 to 46. As before in order to get the MTC magnitude, one should consider in the frequency band 0.1 - 1.0 Hz both the coherence and the phase between the average temperature noise and the neutron noise. The coherence should obviously be large enough, and the phase should be close to ±180 deg since the MTC is a negative reactivity coefficient in the present case. These considerations allow pointing out the frequencies at which the noise estimator should give a correct MTC estimation. Averaging over all these frequencies, one finds an MTC magnitude in the vicinity of 50 pcm/°C, although the coherence for these estimations was sometimes very low (especially for the case where the neutron noise is measured in the fuel channel F04). The coherence was noticed to be much larger when the neutron noise was measured in the fuel channel J12. Furthermore, using different in-core neutron detectors gives slightly different results.

Fig. 42. Comparison between the radial average temperature noise in the core and at the core-exit.
Fig. 43. MTC estimation by using the radial average of the GT signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel D05.
Fig. 44. MTC estimation by using the radial average of the GT signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel F04.
Fig. 45. MTC estimation by using the radial average of the GT signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel F06.
Fig. 46. MTC estimation by using the radial average of the GT signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel J12.
As mentioned in the Introduction, the new MTC noise estimator is biased due to the fact that the neutron noise is measured in one single location, thus measuring the total neutron noise. Formally, the point-kinetic component of the neutron noise should be used while performing the evaluation. As noticed previously when using different in-core neutron detectors, slightly different results are obtained, thus suggesting that the reactor does not exactly behave in a point-kinetic manner. Although it was theoretically demonstrated in Stage 7 [8] that this bias was expected to be rather small, using several in-core neutron detectors simultaneously allows estimating the point-kinetic component of the neutron noise in the core in an approximate manner as follows:

\[
\delta \phi_{pk}(r, t) = \phi_0(r) \frac{\sum_{i=1}^{N} \delta \phi(r_i, t) \phi_0(r_i)}{\int \phi_0^2(r) dr} = \phi_0(r) \frac{\sum_{i=1}^{N} \delta \phi(r_i, t) \phi_0(r_i)}{\sum_{i=1}^{N} \phi_0^2(r_i)}
\]

where \( N \) is the number of available in-core neutron detectors. Eq. (20) is the result of the orthogonality between the static flux \( \phi_0(r) \) and the shape function \( \psi(r, t) \) (see Eq. (4) and Section 2.2 for further details). Such an attempt to estimate the MTC by approximating the point-kinetic response of the reactor is depicted in Fig. 47. It can be seen in such a case that the behaviour of the MTC magnitude is much smoother than while using a single in-core neutron detector and one can therefore consider that the best results are achieved when using the weighted-average of all in-core neutron detectors (approximation of the point-kinetic response).
Fig. 47. MTC estimation by using the radial average of the GT signals on the 7th plane from the top of the core active height and by using the point-kinetic component of neutron noise.
It is also possible to estimate the MTC by using the radial average temperature noise at the core-exit instead of inside the core. For the neutron noise measurement, one can use either all possible in-core neutron detectors respectively or a weighted-average of all detectors (in order to approximate the point-kinetic component of the flux noise). According to the previous calculation of the calibration factor between the radial average of the temperature noise inside the core and at the core-exit, all MTC estimations should be biased low by a factor of roughly 10. As can be seen in Figs. 48 to 52, using the core-exit thermocouples underestimates the MTC by a factor 10. This underestimation is identical when a single in-core neutron detector is used or when the point-kinetic component of the reactor is approximated. Another extremely interesting feature of these estimations is the phase between the core-exit temperature noise and the neutron noise. The phase is not any longer in the vicinity of ±180 deg in the frequency band 0.1 - 1.0 Hz, therefore indicating that one does not and cannot evaluate the MTC any longer in this frequency region by using core-exit thermocouples. As before, using different in-core neutron detectors gives slightly different results, suggesting that the reactor does not exactly behave in a point-kinetic manner.
Fig. 48. MTC estimation by using the radial average of the core-exit thermocouple signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel D05.
Fig. 49. MTC estimation by using the radial average of the core-exit thermocouple signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel F04.
Fig. 50. MTC estimation by using the radial average of the core-exit thermocouple signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel F06.
Fig. 51. MTC estimation by using the radial average of the core-exit thermocouple signals on the 7th plane from the top of the core active height and by using the in-core neutron detector located in the channel J12.
Fig. 52. MTC estimation by using the radial average of the core-exit thermocouple signals on the 7th plane from the top of the core active height and by using the point-kinetic component of neutron noise.
3.4. Analysis of the measurement number 3

The measurement requirements for the measurement number 3, depicted in Fig. 53, were as follows:

- All 29 gamma-thermometers (12 on the lower plane – plane 7 –, 10 on the upper plane – plane 3 –, and 9 in the fuel assembly J10) should have been monitored;
- 3 in-core neutron detectors should have been inserted successively;
- The measurement length should have been approximately equal to 3x20 min, i.e. 1 hour.

The purpose of this measurement was actually to improve the statistics of the signals by having a much longer measurement recording. The limiting factors in the usual way of performing an MTC noise measurement are the in-core neutron detectors, which cannot be inserted in the core for more than typically 20-30 min before their efficiency changes significantly. A way of circumventing this problem is actually to insert several detectors successively in the core, detectors located in the same region of the core. Since the space-dependence of the neutron noise for randomly distributed noise sources is radially weakly space-dependent, one can consider that these 3 detectors are measuring the neutron noise almost at the same location. Concatenating the time signals of these 3 detectors together would have made a time signal three times longer, and would have therefore improved the statistics and the accuracy of the MTC noise estimations.

This last measurement was unfortunately not exploitable since the 3 in-core neutron detectors were inserted simultaneously, and not successively.

Fig. 53. Radial location of the detectors on the left hand-side, and axial location on the right hand-side, with the fuel assembly J10 axially fully instrumented with gamma-thermometers.
3.5. Conclusions and future work

As for Stage 7, the MTC was correctly estimated when using the new MTC noise estimator. This estimator relies on the neutron noise measurement in one single point of the core and on the radial core average moderator temperature noise measured inside the core. It was also demonstrated in this study that using simultaneously several in-core neutron detectors improves the MTC estimation since the point-kinetic response of the reactor can thus be approximately determined from these detectors. This is due to the fact that the new MTC noise estimator still relies on a point-kinetic behaviour of the reactor. If one tries to use a radial average of the core-exit thermocouples instead of a radial average of the in-core thermocouples, it becomes unfortunately impossible to estimate the MTC correctly since the core-exit moderator temperature noise is fundamentally different from the in-core moderator temperature noise. Consequently, the new MTC noise estimator could only be applied to PWRs equipped with in-core thermocouples (in the case of Ringhals-2, these in-core thermocouples are actually GTs). Finally, it was noticed that using ex-core neutron detectors instead of in-core neutron detectors allows improving the statistics of the signals by having longer measurements, but do not improve drastically the accuracy of the MTC noise estimation.

The reason for this is the fact that in most cases the mean value of the signals of the GTs was not removed before the A/D conversion (most of the signals coming from temperature detectors – GTs or core-exit thermocouples – were recorded without DC removal). The DC removal would have improved significantly the MTC estimation.

A phenomenon that was actually pointed out in Stage 7 and that is still not completely understood is the frequency dependence of the MTC noise estimator in the frequency band 0.1 - 1.0 Hz. Several factors could render the MTC noise estimator frequency-dependent such as the effect of the different time constants of the detectors, or the effect of the Doppler coefficient. Further investigations are required in this respect. Wavelet analysis could allow getting a smoother frequency dependence of the MTC noise estimator, either by calculating the APSDs and CPSDs used in the MTC noise estimator from wavelet analysis, or by deriving directly a wavelet-based MTC noise estimator. Wavelet techniques could thus be investigated at a later stage.

4. CALCULATION OF THE NEUTRON NOISE INDUCED BY SHELL-MODE CORE-BARREL VIBRATIONS IN A 1-D 2-GROUP 2-REGION SLAB REACTOR

4.1. Introduction

The ex-core neutron noise, induced by core-barrel vibrations have long been used to diagnose both beam-mode and shell-mode vibrations. In connection with the study of in-core neutron noise induced by fluctuating system boundaries, [18], it has been realized that core-barrel vibrations might lead also to in-core noise. This is especially useful for the diagnostics of shell-mode vibrations in Westinghouse reactors because all ex-core detectors carry the same information, due to the 90° spacing. Hence it is not possible to determine both the vibration amplitude and the direction from the detector signals. In particular it is not possible to find out if the reason for a change in the signal amplitude is due to the change in the vibration amplitude or the direction of the vibrations.
For this reason we have started including the in-core detector signals in the analysis of core-barrel vibrations in the collaborative research project between Ringhals and the Department of Reactor Physics at Chalmers University of Technology ([1] -[8]). However, confirmation of the theory as well as use of the results for diagnostics was hindered by the low number of in-core detectors. In particular, during evaluation of the only available measurement in Ringhals-3 when both in-core and ex-core detectors were available, in order to have a consistent interpretation, it was necessary to assume that ex-core and in-core detectors lying on the same azimuthal position have opposite phase.

This out-of-phase behaviour was contradicting the simple theory that was used so far. However, in that theory the in-core noise was only calculated in a non-reflected system. One cannot exclude the possibility that the core-barrel vibrations, if treated in a reflected system, will lead to another structure of the neutron noise than in a bare system. For instance, the outbound movement of the boundary in the unreflected system means some multiplicative material added outside the static boundary, which then would lead to both an increase of reactivity and, in addition, to a local increase of the neutron flux. Both components would be in phase with the signal of the ex-core detectors. However, in a reflected system, an outbound movement of the core boundary means a decreasing reflector thickness, and, under certain conditions, also possibly a decrease of reactivity. Hence, it is not obvious whether such a movement will lead to the increase of the in-core flux or not.

The purpose of this report is to investigate the in-core noise induced by core-barrel vibrations in reflected systems by the extension of the model used earlier. In order to be able to handle a reflected system, it was necessary to use two-group theory. To compensate for the complications that this extension incurs, the description was reduced to one spatial dimension. Density variations, induced by the 1-D shell mode vibrations, were also accounted for. The noise was calculated in the adiabatic approximation. This made it possible to handle the problem analytically throughout, with a numerical evaluation of the final formulas.

4.2. Description of the reactor model

A one-dimensional model of a reflected reactor is selected for this study with a central core and outer reflectors placed symmetrically around the core (Fig. 54). Two-group diffusion theory will be used to calculate the noise, with corresponding cross sections and diffusion constants in the fast, 1, and thermal, 2, group, respectively. One averaged group of delayed neutrons was used in the dynamic calculations. The shell-mode vibrations will be modelled by simultaneous, symmetric vibrations of the core boundary around the static position \( x = \pm b \). Hence both the static and the dynamic case will be symmetric around the origin, and this simplifies the solution of the problem.

\[
\begin{array}{c|c|c|c}
\phi_{1,2}^f(x) & \phi_{1,2}^c(x) & \phi_{1,2}^r(x) \\
\hline
-a & -b & 0 & b & a
\end{array}
\]

\textbf{Fig. 54.} 1-D Core with reflector

Since the original idea was to see if there can, at least under certain circumstances, exist an out-of-phase behaviour between in-core and ex-core (ex-vessel) detectors, the
boundary condition at the outer boundary of the system was chosen to be the no incoming current condition, which can be expressed by the logarithmic flux derivative and the extrapolation length, see below. This way the scalar flux does not vanish at the system boundary, and the ex-core detector signals were chosen to be equal to the flux at the boundary.

4.3. The static case

4.3.1. Boundary and interface conditions

To start we define two functions in the core: $\phi^{c, 1}_{1, 2}(x), -b \leq x \leq b$, and two functions in the reflector: $\phi^{r, 1}_{1, 2}(x), -a \leq x \leq -b, b \leq x \leq a$. The subscript indicates if it is the fast, 1, or the thermal, 2, energy group. The superscript $r$ stands for the reflector and $c$ for the core. Because of the symmetry of the problem we have first of all:

$$\begin{cases}
\phi^{c, 2}_{1, 2}(-x) = \phi^{r, 2}_{1, 2}(x), -b \leq x \leq b \\
\phi^{r, 2}_{1, 2}(-x) = \phi^{c, 2}_{1, 2}(x), -a \leq x \leq -b, b \leq x \leq a
\end{cases} \tag{21}$$

This gives us the possibility to set up the problem only for $x \geq 0$.

The boundary and interface conditions used are described below in equation (22) and (23)

\[
\begin{align*}
\phi^{r}_{1}(x)|_{x = b_{+}} &= \phi^{c}_{1}(x)|_{x = b} \\
\phi^{r}_{2}(x)|_{x = b_{+}} &= \phi^{c}_{2}(x)|_{x = b} \\
D^{r}_{1}\frac{\partial \phi^{r}_{1}(x)}{\partial x}|_{x = b_{+}} &= D^{c}_{1}\frac{\partial \phi^{c}_{1}(x)}{\partial x}|_{x = b} \\
D^{r}_{2}\frac{\partial \phi^{r}_{2}(x)}{\partial x}|_{x = b_{+}} &= D^{c}_{2}\frac{\partial \phi^{c}_{2}(x)}{\partial x}|_{x = b} \tag{22}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \phi^{r}_{1}(x)}{\partial x}|_{x = a} &= -\frac{1}{d^{1}}\phi^{c}_{1}(x)|_{x = a} \\
\frac{\partial \phi^{r}_{2}(x)}{\partial x}|_{x = a} &= -\frac{1}{d^{2}}\phi^{c}_{2}(x)|_{x = a} \tag{23}
\end{align*}
\]

Here $d_{1}$ and $d_{2}$ are the extrapolation lengths. They are defined through the cross-sections and diffusion constants in the reflector.

\[
\begin{align*}
\begin{cases}
d_{1} = 0.71\frac{1}{\Sigma_{tr, 1}} = 0.71 \cdot 3D^{c}_{1} \\
d_{2} = 0.71\frac{1}{\Sigma_{tr, 2}} = 0.71 \cdot 3D^{c}_{2}
\end{cases} \tag{24}
\end{align*}
\]

These are all the conditions we need to solve the problem and we are free to normalize the flux.
4.3.2. Equations in the reflector

The diffusion equations for the fast and thermal fluxes in the reflector region, were no fission occurs are as follows:

\[
\begin{align*}
D_1^r \frac{d^2}{dx^2} \phi_1^r(x) - (\Sigma_{a,1}^r + \Sigma_R^r) \phi_1^r(x) &= 0 \\
D_2^r \frac{d^2}{dx^2} \phi_2^r(x) - \Sigma_{a,2}^r \phi_2^r(x) + \Sigma_R^r \phi_1^r(x) &= 0
\end{align*}
\]

(25)

Now, the subscript for the cross sections displays which type of reaction it is, \(a\) for absorption, \(R\) (removal) for scattering from the fast to the thermal group and \(f\) for fission in the core region. The superscript \(r\) stands for the reflector and later \(c\) for the core. The general solutions of these coupled differential equations are easily derived and simplified by using (21). The constants will be determined later when solving the core equation an using the boundary and interface conditions. Although, there are six constants and six conditions one of the constants will be undetermined since an equation for the criticality appear from the six conditions. Though, we have to deal with the large exponents due to the large size of a real reflector (\(a \sim 300\) cm and \(e^{10}\)). By multiplying the solution with \(e^{-b}\) and use new integration constants, the numerical values become smaller:

\[
\begin{align*}
\phi_1^r(x) &= \alpha_3 e^{\kappa_1(|x| - \tilde{b})} + \alpha_4 e^{-\kappa_1(|x| - \tilde{b})} \\
\phi_2^r(x) &= \alpha_5 e^{\kappa_2(|x| - \tilde{b})} + \alpha_6 e^{-\kappa_2(|x| - \tilde{b})} + \frac{\Sigma_R^r}{D_2^r(\kappa_2^2 - \kappa_1^2)} \phi_1^r(x)
\end{align*}
\]

(26)

where \(\tilde{b} = \frac{a - b}{2}\)

Here the following notations are used:

\[
\kappa_1 = \frac{1}{L_1^r} = \sqrt[!]{\frac{\Sigma_1^r}{D_1^r}}, \quad \kappa_2 = \frac{1}{L_2^r} = \sqrt[!]{\frac{\Sigma_{a,2}^r}{D_2^r}} \\
\Sigma_1^r = \Sigma_{a,1}^r + \Sigma_R^r
\]

(27)

4.3.3. Equations in the core

The equations in the core are a little bit more complicated due to the fact that they involve fission terms that are not present in the reflector region. This means that the fast and thermal equations are more coupled than in the reflector region. We assume that all fission neutrons are fast but one could also assume that some part of them are thermal. Then one would have to add a fission term in the equation for the thermal ones, using the spectrum of the fission neutrons.
When solving this equation system we try with the following form of the fluxes.

\[
\begin{align*}
\phi_1^c(x) &= \phi_{1,0}^c e^{\lambda x} \\
\phi_2^c(x) &= \phi_{2,0}^c e^{\lambda x}
\end{align*}
\]  

(29)

With this inserted into (28), we arrive with a matrix equation:

\[
\begin{bmatrix}
D_1^c \lambda^2 & 0 \\
0 & D_2^c \lambda^2
\end{bmatrix}
\begin{bmatrix}
\phi_{1,0}^c \\
\phi_{2,0}^c
\end{bmatrix}
+ 
\begin{bmatrix}
-S_1^c & \frac{1}{k} \nu \Sigma_{f,2}^c \\
\Sigma_R^c & -S_{a,2}^c
\end{bmatrix}
\begin{bmatrix}
\phi_{1,0}^c \\
\phi_{2,0}^c
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(30)

with

\[
\Sigma_1^c = \Sigma_{a,1}^c + \Sigma_R^c - \frac{1}{k} \nu \Sigma_{f,1}^c
\]

(31)

and if

\[
det
\begin{bmatrix}
-S_1^c - D_1^c \lambda^2 & \frac{1}{k} \nu \Sigma_{f,2}^c \\
\Sigma_R^c & -S_{a,2}^c - D_2^c \lambda^2
\end{bmatrix}
= 0
\]

(32)

there exist a solution to the equations. And the eigenvalues, \(\lambda\), can be determined and expressed in terms of the cross sections and diffusion constants of the two energy groups as:

\[
(\lambda^2)_{1,2} = \frac{(D_1^c \Sigma_{a,2}^c + D_2^c \Sigma_1^c) \pm \sqrt{(D_1^c \Sigma_{a,2}^c - D_2^c \Sigma_1^c)^2 + \frac{4}{k} \nu \Sigma_{f,2}^c \Sigma_R^c D_1^c D_2^c}}{2D_1^c D_2^c}
\]

(33)

This gives four eigenvalues which appear in two pairs:
We also have a relation between and from equation 30 reading as

\[ \lambda_{3,4} = \pm \mu = \pm \sqrt{\frac{(D_1^c \Sigma_{a,2}^c - D_2^c \Sigma_1^c)^2 + \frac{4}{k} \nu \Sigma_{f,2}^c D_1^c D_2^c}{2D_1^c D_2^c}} \]

(34)

This gives us two constants:

\[ C_1 = \frac{\Sigma^c_R}{\Sigma^c_{a,2} - D_2^c \lambda^2} \]

\[ C_2 = \frac{\Sigma^c_R}{\Sigma^c_{a,2} + D_2^c \mu^2} \]

(36)

Together with the eigenvalues this gives us the solution of the fast and thermal flux in the core although we have to determine the criticality of the system.

\[
\begin{align*}
\phi_1^c(x) &= \alpha_1' e^\eta x + \tilde{\alpha}_1 e^{-\eta x} + \alpha_2' e^{i\mu x} + \tilde{\alpha}_2 e^{-i\mu x} \\
\phi_2^c(x) &= C_1(\alpha_1' e^\eta x + \tilde{\alpha}_1 e^{-\eta x}) + C_2(\alpha_2' e^{i\mu x} + \tilde{\alpha}_2 e^{-i\mu x})
\end{align*}
\]

(37)

Due to the symmetry conditions expressed by (21), the above can be simplified to:

\[
\begin{align*}
\phi_1^c(x) &= \alpha_1 \cosh \eta x + \alpha_2 \cos \mu \\
\phi_2^c(x) &= \alpha_1 C_1 \cosh \eta x + \alpha_2 C_2 \cos \mu x
\end{align*}
\]

(38)

with \( \alpha_1 = \alpha'_1 = \tilde{\alpha}_1 \) and \( \alpha_2 = \alpha'_2 = \tilde{\alpha}_2 \)

Now (26) and (38) are the general solutions in the reflector and the core, and to determine the constants and the criticality the boundary and interface conditions have to be used.

**4.3.4. Critical equation**

Two of the constants are already determined with the symmetry so six constants and the eigenvalue (criticality) are to be determined. To fully determine the eigenvalue and the constants we need a normalization condition, which will be done by setting the fast flux to
unity, in the centre of the core. Using (22) and (23) we arrive at a matrix equation $A^*\alpha = 0$ where the matrix $A$ is:

$$
\begin{bmatrix}
\cosh \eta b & \cos \mu b & -\varepsilon_1 & -\frac{1}{\varepsilon_1} & 0 & 0 \\
C_2 \cosh \eta x & C_1 \cos \mu x & \varepsilon_1 \kappa & \frac{\kappa}{\varepsilon_1} & -\varepsilon_2 & -\frac{1}{\varepsilon_2} \\
0 & 0 & -\left(\kappa_1 + \frac{1}{d_2}\right) \kappa_1 & \left(\frac{\kappa_1 - 1}{d_2}\right) \frac{\kappa}{\delta_1} & \left(\kappa_2 + \frac{1}{d_2}\right) \delta_2 & \left(\frac{1}{d_2} - \kappa_2\right) \frac{1}{\delta_2} \\
0 & 0 & \left(\kappa_1 + \frac{1}{d_1}\right) \delta_1 & \left(\frac{\kappa_1 - 1}{d_1}\right) \frac{1}{\delta_1} & 0 & 0 \\
\eta \sinh \eta b & -\mu \sin \mu b & -\kappa_1 D_1^r \varepsilon_1 & \frac{\kappa_1 D_1^r}{D_1^r} & 0 & 0 \\
C_2 \eta \sinh \eta b & C_1 \mu \sin \mu b & \frac{\kappa_1 D_2^r \varepsilon_1}{D_2^r} & \frac{\kappa_1 D_2^r}{D_2^r} & -\kappa_2 D_2^r \varepsilon_2 & \frac{\kappa_2 D_2^r}{D_2^r} \\
\end{bmatrix}
$$

(39)

Here the constants used to simplify the matrix are

$$
\varepsilon_1 = e^{(b - \hat{b}) \beta_1^r}, \quad \varepsilon_2 = e^{(b - \hat{b}) \beta_2^r}, \\
\delta_1 = e^{(a - \hat{b}) \beta_1^r}, \quad \delta_2 = e^{(a - \hat{b}) \beta_2^r}, \quad \kappa = \frac{-\Sigma^r}{(\kappa_2^2 - \kappa_1^2) D_2^r}
$$

(40)

$n$ and $u$ were defined in (34). The critical equation for determining $k_{\text{eff}}$ is:

$$
f(k) \equiv \det(A) = 0
$$

(41)

Using Mathematica one arrives at an equation for $k_{\text{eff}}$: 
which consists of some $\tan$ and $\tanh$ functions and must be solved numerically. The constants in this equation are just some expressions involving other known terms as the extrapolation lengths and some exponential terms which are defined in (40).

The solution, which is physical, is the on with the largest $k_{\text{eff}}$. See Fig. 55. By using that value in (34) the eigenvalues, $\eta$ and $\mu$ are determined. The structure of the critical equation plotted as a function of the inverse variable is displayed in Fig. 55. It shows that the $k$-function consists of infinitely many branches separated by the asymptotes:

$$f(k_{\text{eff}}) = \left( \frac{\Sigma_{a,2}^c}{D_2^c} - \eta_1^2 \right) \left( \eta \tan b \eta + h_1 \right) - \left( \mu \tan b \mu - \left( \frac{\mu^2 + \Sigma_{a,2}^c}{D_2^c} \right) * h_{12} + h_2 \right)$$

where $\eta \equiv \eta(k_{\text{eff}})$, $\mu \equiv \mu(k_{\text{eff}})$ and $h_1$, $h_2$ and $h_{12}$ are constants which consist of some $\tan$ and $\tanh$ functions and must be solved numerically. The constants in this equation are just some expressions involving other known terms as the extrapolation lengths and some exponential terms which are defined in (40).

The solution, which is physical, is the on with the largest $k_{\text{eff}}$. See Fig. 55. By using that value in (34) the eigenvalues, $\eta$ and $\mu$ are determined. The structure of the critical equation plotted as a function of the inverse variable $\lambda \equiv 1/k$ is displayed in Fig. 55. It shows that the $k$-function consists of infinitely many branches separated by the asymptotes:

$$\lambda_n = \frac{(D_1^c \mu_n^2 + (\Sigma_{a,1}^c + \Sigma_{R}^c))(D_2^c \mu_n^2 + \Sigma_{a,2}^c)}{\sqrt{\frac{\Sigma_{f,1}^c}{2} (D_2^c \mu_n^2 + \Sigma_{a,2}^c) + \sqrt{\Sigma_{f,2}^c + \Sigma_{R}^c}}}; \quad n = 0, 1, 2, \ldots$$

A minimal bound, $\lambda_{\text{min}}$, may be defined by $\mu(\lambda_{\text{min}}) = 0$, which gives

![Fig. 55. Plot of the k-function](image-url)
Then inserting them into the matrix equation and solving it gives the integrations constants. Since this gives us six constants that are linearly dependent we use the normalization condition \( \phi_i(x=0) = 1 \) to fully determine all six constants. The fast and thermal fluxes are now fully determined and are displayed in Fig. 56 with a core radius of 161.25 cm and a reflector radius of 279.5 cm. The flux from a numerical simulator [19], developed at our department is also plotted and as one sees the analytical solution agree very well with the solution from the simulator. This together with the reflector peaks of the thermal flux indicates the correctness of the solution. The size of the reactor is supposed to represent a real reactor. All the cross-sections are taken from SIMULATE-3. Although SIMULATE-3 started with a 3-D critical system the 1-D system is super-critical since it is less leakage in a 1-D system. The value for \( k_{eff} \) is 1.00146 from the simulator and the same for the analytical calculation done in this report. Although, there is a mismatch in the sixth decimal. Note that the fast flux is about ten times larger than the thermal.

\[
\lambda_{\text{min}} = \frac{(\Sigma_{a,1}^c + \Sigma_{a,2}^c)\Sigma_{a,2}^c}{(\nu\Sigma_{f,1}^c\Sigma_{a,2}^c + \nu\Sigma_{f,2}^c\Sigma_{R}^c)}
\]

Fig. 56. Fast and Thermal flux form semi-analytic calculations and from SIMULATE-3. \( k=1.00146 \) in both cases.

To really visualise the reflector peaks and the ratio between the fast and thermal fluxes they are plotted together in Fig. 57. Only the region close to the boundary between the core and
reflector is displayed. The peak of the thermal flux in the reflector arises from the slowing down of the fast neutrons to thermal ones that occurs in the reflector. Notice also that the system is critical in this figure compared to Fig. 56, this is done by rescaling the fission cross-sections and using the same parameters as in the super critical system.

4.3.5. Adjoint flux

What we want to investigate is how a vibration of the core-barrel effects the detector signal, i.e. perturb the thermal flux. To calculate this noise we need to calculate the adiabatic flux, and to normalize that we need the static adjoint flux. The adjoint flux and the direct flux have the same eigenvalue, same k, so we do not need to determine the criticality once more. Also the interface and boundary conditions are the same. The adjoint equations are similar to the direct ones, so the idea of solving is the same. The adjoint equations in the reflector are:

\[
\begin{align*}
D_1^r \frac{d^2}{dx^2} \phi_1^{+r}(x) - \left( \Sigma_{a,1}^r + \Sigma_{s,1}^r \right) \phi_1^{+r}(x) + \Sigma_{s,1}^r \phi_2^{-r}(x) &= 0 \\
D_2^r \frac{d^2}{dx^2} \phi_2^{-r}(x) - \Sigma_{a,2}^{-r} \phi_2^{-r}(x) &= 0
\end{align*}
\]

(45)

and in the core:

\[
\begin{align*}
D_1^c \frac{d^2}{dx^2} \phi_1^{+c}(x) - (\Sigma_{a,1}^c + \Sigma_{s,1}^c) \phi_1^{+c}(x) + \frac{1}{k} \nabla \Sigma_{f,1}^c \phi_1^{+c}(x) + \Sigma_{s,1}^c \phi_2^{+c}(x) &= 0 \\
D_2^c \frac{d^2}{dx^2} \phi_2^{+c}(x) - \Sigma_{a,2}^{+c} \phi_2^{+c}(x) + \frac{1}{k} \nabla \Sigma_{f,2}^c \phi_1^{+c}(x) &= 0
\end{align*}
\]

(46)

The solutions for the adjoint fluxes in the reflector are:
For the core the solution looks like

\[
\begin{align*}
\phi_2^+(x) &= \alpha_5 e^{-\lambda_2|x| - \lambda_6} + \alpha_6 e^{-\lambda_3|x| - \lambda_7} \\
\phi_1^+(x) &= \alpha_3 e^{\lambda_1|x| - \lambda_2} + \alpha_4 e^{\lambda_2|x| - \lambda_3} - \frac{\Sigma_R'}{D_1'(\kappa_2 - \kappa_1^2)} \phi_1'(x)
\end{align*}
\]

(47)

Compared with the direct solution, (26) and (38), we see that the fast and thermal flux in some sense have changed places. Though, the constants are somewhat different. To see the difference between the direct and adjoint fluxes they are plotted in Fig. 58. The system has a core radius of 20 cm and a reflector radius of 40 cm, but it is still critical. Showing this smaller system makes the difference between the direct and adjoint fluxes clearer.

\[
\begin{align*}
\phi_2^+(x) &= \alpha_1^+ \cosh \eta x + \alpha_2^+ \cos \mu \\
\phi_1^+(x) &= \alpha_3^+ \cosh \eta x + \alpha_4^+ \cos \mu x
\end{align*}
\]

(48)

\[
C_1^+ = \frac{\Sigma_R^c}{(\Sigma_1^c - D_1^c \eta^2)}, \quad C_2^+ = \frac{\Sigma_R^c}{(\Sigma_1^c + D_1^c \mu^2)}
\]

Compared with the direct solution, (26) and (38), we see that the fast and thermal flux in some sense have changed places. Though, the constants are somewhat different. To see the difference between the direct and adjoint fluxes they are plotted in Fig. 58. The system has a core radius of 20 cm and a reflector radius of 40 cm, but it is still critical. Showing this smaller system makes the difference between the direct and adjoint fluxes clearer.

Fig. 58. Direct and Adjoint flux
4.4. The dynamic case

We now proceed with a dynamic model that describes the response of the originally static system to time-dependent perturbation. Assuming one average group of delayed neutron precursors, \( C \), in addition to two prompt neutron groups, we have for the slab reactor

\[
\frac{1}{v_1} \frac{\partial \phi_1}{\partial t} = \frac{\partial}{\partial x} D_1 \frac{\partial \phi_1}{\partial x} - (\Sigma_{a,1} + \Sigma_{s, 1}) \phi_1 + (1 - \beta)(\nu \Sigma_{f,1} \phi_1 + \nu \Sigma_{f, 2} \phi_2) + \lambda C
\]

\[
\frac{1}{v_2} \frac{\partial \phi_2}{\partial t} = \frac{\partial}{\partial x} D_2 \frac{\partial \phi_2}{\partial x} - \Sigma_{a,2} \phi_2 + \Sigma_{s, 1} \phi_1
\]

\[
\frac{\partial C}{\partial t} = -\lambda C + \beta(\nu \Sigma_{f,1} \phi_1 + \nu \Sigma_{f, 2} \phi_2)
\]

In contrast to the static system (25) and (28) now the unknown functions and cross sections are space and time dependent

\[
\phi_1 \equiv \phi_1(x, t); \quad D_1 \equiv D_1(x, t); \quad \Sigma_{a,1} \equiv \Sigma_{a,1}(x, t); \ldots
\]

\[
\phi_2 \equiv \phi_2(x, t); \quad D_2 \equiv D_2(x, t); \quad \Sigma_{a,2} \equiv \Sigma_{a,2}(x, t); \ldots
\]

\[
C \equiv C(x, t)
\]

We model the shell mode vibrations by letting the core boundary, \( \delta b(t) \), oscillate around the static position, \( b \), at both sides of the core in a symmetrical manner:

\[
b(t) = b + \delta b(t); \quad \langle \delta b(t) \rangle = 0
\]

In addition, we assume the value of the cross sections in the core to be proportionally modified

\[
\Sigma^c_{x}(x, t) = \Sigma^c_{x, static} \cdot \frac{b}{b + \delta b(t)} \quad D^c_{x}(x, t) = D^c_{static} \cdot \left( \frac{b}{b + \delta b(t)} \right)^{-1}
\]

whereas the value of the cross sections in the reflector is assumed to be unaffected by the vibrations. It should be noted here that because of the moving core boundary all the cross sections must be regarded as time and space dependent even in the reflector.

Instead of solving equations (49) directly we are going to derive properties of the perturbed system by using the adiabatic approximation. To this order we start by factorising the unknown fluxes into an amplitude factor, \( P(t) \), and a shape function, \( \psi(x, t) \), as follows

\[
\begin{align*}
\phi_1(x, t) &= P(t) \cdot \psi_1(x, t) \\
\phi_2(x, t) &= P(t) \cdot \psi_2(x, t)
\end{align*}
\]

\[
\begin{align*}
P(0) &= 1 \\
\psi_{1, 2}(x, 0) &= \phi^\text{static}_{1, 2}(x)
\end{align*}
\]

Since we have introduced three new quantities instead of the two unknown functions, \( \phi_1 \) and \( \phi_2 \), we need to impose an additional constraint that normally reads as [20]

\[
\int_{-a}^{a} \left[ \frac{1}{v_1} \phi^+_1(x) \psi_1(x, t) + \frac{1}{v_2} \phi^+_2(x) \psi_2(x, t) \right] dx = \text{const}
\]

Here, \( \phi^+_i(x) \) denotes static adjoint fluxes. Usually, a derivation of the kinetic equations, details of which one can find in [20], goes as follows. One puts the factorization (53) into...
(49), multiplies each equation by the corresponding static adjoint flux, then subtracts the static equations, and finally, integrates over the reactor volume. This yields in the end for the amplitude factor, \( P(t) \), the equations:

\[
\begin{align*}
\frac{dP(t)}{dt} &= \frac{\rho(t) - \beta}{\Lambda(t)} P(t) + \lambda c(t) \\
\frac{dc(t)}{dt} &= \frac{\beta}{\Lambda(t)} P(t) - \lambda c(t)
\end{align*}
\]

The new quantities that appear in (55) involve an arbitrary factor, \( F(t) \), that is usually defined as

\[
F(t) \equiv \int_{-a}^{a} \phi_{0,1}^+(x) \cdot \left[ \nu \Sigma f,1 \psi_1(x, t) + \nu \Sigma f,2 \psi_2(x, t) \right] dx
\]

Then the prompt neutron generation time reads as

\[
\Lambda(t) = \frac{1}{F(t)} \int_{-a}^{a} \left[ \frac{1}{\nu_1} \phi_{0,1}^+(x) \psi_1(x, t) + \frac{1}{\nu_2} \phi_{0,2}^+(x) \psi_2(x, t) \right] dx
\]

The reactivity term becomes

\[
\rho(t) = \frac{1}{F(t)} \left\{ \int_{-a}^{a} \phi_{0,1}^+ \cdot \left[ \Delta(\nu \Sigma f,1) \psi_1 + \Delta(\nu \Sigma f,2) \psi_2 \right] dx + \right. \\
+ \left. \int_{-a}^{a} \Delta(\Sigma r) [\phi_{0,2}^+ - \phi_{0,1}^+] \psi_1 dx - \int_{-a}^{a} [\Delta(\Sigma a,1) \phi_{0,1}^+ \psi_1 + \Delta(\Sigma a,2) \phi_{0,2}^+ \psi_2] dx \right\}
\]

Here, \( \Delta \) denotes the fluctuation in a corresponding cross section, for example

\[
\Delta(\nu \Sigma f,1) \equiv \nu \Sigma f,1(x, t) - \nu \Sigma f,1(x)
\]

Finally, the precursor density, \( c(t) \), reads as

\[
c(t) = \frac{1}{\Lambda(t) F(t)} \int_{-a}^{a} \phi_{0,1}^+(x) C(x, t) dx
\]

It should be noted here that \( \beta \), which is involved in the kinetic equations (35), depends on time \( t \) in the most general case. But our model assumes that both prompt fission and delayed neutrons are born in the fast group only. It follows from the exact definition, [20], that \( \beta \) is time independent and equals to the static delayed neutron fraction under this assumption.

Our next step is to linearize the kinetic equations (55) in order to obtain the zero reactor transfer function for our two-group model. The standard linearization technique starts with splitting quantities into a steady part plus a small deviation:

\[
\rho(t) = \rho_0 + \delta \rho(t); \quad F(t) = F_0 + \delta F(t); \quad \Lambda(t) = \Lambda_0 + \delta \Lambda(t)
\]
One then puts this into the original equations (55), and neglects second-order terms. Here

\[
\begin{align*}
F_0 &= \int_{a}^{b} \phi_{0,1}(x) \cdot \left[ \nu \Sigma,1 \phi_{0,1}(x) + \nu \Sigma,2 \phi_{0,2}(x) \right] dx \\
\Lambda_0 &= \frac{1}{F_0} \int_{-a}^{a} \left[ \frac{1}{v_1} \phi_{0,1}(x) \phi_{0,1}(x) + \frac{1}{v_2} \phi_{0,2}(x) \phi_{0,2}(x) \right] dx
\end{align*}
\]  

(62)

In the end one arrives at the following formula in the frequency domain:

\[
\delta P(\omega) = G_0(\omega) \cdot \delta \rho(\omega)
\]  

(63)

Here, \(G_0(\omega)\) is a transfer function that is similar to the ordinary (one-group) zero reactor transfer function:

\[
G_0(\omega) = \frac{1}{i\omega \left( \Lambda_0 + \frac{\beta}{\lambda + i\omega} \right)}
\]  

(64)

with an energy and space averaged constant, \(\Lambda_0\), defined in (62). In the plateau region, \(\lambda < \omega < \beta / \Lambda_0\) (see Fig. 59), an approximate equality holds:

\[
|G_0(\omega)| \approx \frac{1}{\beta}
\]  

(65)

Equation (63) in conjunction with (65) allows us to derive both a very important and useful relationship in the plateau region, namely:

\[
\delta P(\omega) = \frac{1}{\beta} \delta \rho(\omega) \Rightarrow \delta P(t) = \frac{1}{\beta} \delta \rho(t)
\]  

(66)

The importance of the above formula stems from the fact that \(\delta \rho(t)\) may be relatively easily evaluated by calculating a corresponding eigenvalue problem:

\[
\delta \rho(t) = 1 - \frac{1}{k(t)}
\]  

(67)

Here \(k(t)\) depends on time \(t\) parametrically through the time-dependence of the corresponding cross sections. A straightforward linearization of (53) gives us

\[
\begin{align*}
\delta \phi_1(x, t) &= \delta P(t) \cdot \phi_{0,1}(x) + \delta \psi_1(x, t) \\
\delta \phi_2(x, t) &= \delta P(t) \cdot \phi_{0,2}(x) + \delta \psi_2(x, t)
\end{align*}
\]  

(68)

The first term on the right-hand-side is called the point reactor term whereas the second one is referred to as the space-dependent term.

The space-dependent terms may be relatively easily evaluated through the adiabatic approximation:
Here, $\psi_{1,2}(x,t)$ are the positive eigenfunctions for the eigenvalue problem (28) with the cross sections corresponding to the time instance $t$ (i.e. the momentary core boundary $b(t)$). Eq. (68) combined with (69) and (66) gives, finally, the following approximation:

$$
\begin{align*}
\delta \psi_1(x,t) &\approx \delta \psi_{1}^{ad}(x,t) \equiv \psi_1^{ad}(x,t) - \phi_{0,1}(x) \\
\delta \psi_2(x,t) &\approx \delta \psi_{2}^{ad}(x,t) \equiv \psi_2^{ad}(x,t) - \phi_{0,2}(x)
\end{align*}
$$

(69)

In our numerical calculations we use the following parameters

$$
\beta = 0.00535; \quad \lambda = 0.08510; \quad \Lambda_0 = 2.0776 \cdot 10^{-5}
$$

(71)

that correspond to a plateau region of $10^{-1} < \omega < 10^{2}$. The shell mode vibrations are expected to be around 125 rad/s, which is within the plateau region.

Fig. 59. Zero power transfer function
4.5. Numerical work

Now we have everything we need to calculate the effect on the flux due to a vibration of the core-barrel. The vibrations are assumed to change the volume of the core but not its mass. Hence, the density of the material present, changes due to the larger (or smaller) size of the core and this will affect all cross sections as well as the diffusion constants.

The new cross sections and diffusion constants in the core are:

\[
\Sigma^c_x(x, t) = \Sigma^c_{x, \text{static}} \frac{b}{b + \delta b}
\]

\[
D^c(x, t) = D^c_{\text{static}} \left( \frac{b}{b + \delta b} \right)^{-1}
\]  

(72)

Here $\delta b$ is the change of the boundary between the core and the reflector. In the reflector on the other hand, the density is assumed to be the same so all cross sections and diffusion constants are the same. By this we assume that some water is removed (added) from the reflector when its size is smaller (larger). In other words we assume that it is impossible to compress the water. Now, it is possible to calculate a new $k_{\text{eff}}$ and a new flux $\phi_{\text{adiabatic}}(x)$ . And by using the new $k_{\text{eff}}$, $\rho(t)$ can be calculated. Then one can use (70) to calculate the new flux with its time-dependent perturbation. Out of this it is possible to determine the phase-behaviour between in-core and ex-core positions (detectors), due to this perturbation.

4.6. Results and Discussion

When investigating the phase behaviour one has to estimate the size of the core barrel vibration. We assume that it is in the order of mm if the core has a size of several metres. The core used in SIMULATE-3, for calculation of the reactor parameters, is Ringhals 4 and it has a core radius of 152.0 cm and the outer radius of the reflector is 166.3 cm (the reflector is 14.3 cm thick). The small thickness of the reflector is due to the fact that SIMULATE-3 only uses one node for modelling the reflector. But on the other hand we can extend the reflector to a radius of 279.5 cm and the core radius is actually 161.25 cm in the 1-D model used by our simulator. Using this we calculated $k_{\text{eff}} = 1.00146$ and in our analytical model described above we also received $k_{\text{eff}} = 1.00146$, as one can see they match perfectly.

However, it is possible to choose any size of the system. One just has to adjust the fission cross sections so that the system becomes critical. To display the features of the system we choose a small system and a large unphysical movement of the core boundary. By using a core radius of 20 cm, a reflector radius of 40 cm and a displacement of the core of 2 cm we see that there is an out-of-phase behaviour (shaded area, Fig. 61) between the whole core and the ex-core detector. Since this change of the boundary, is quite large, the
factor $\rho/\beta$ is also large meaning that the point reactor term, i.e. the first term on right hand side in (70), will contribute to the perturbed flux together with the space-dependent term.

For the ex-core value we us the thermal flux at the outer edge of the reflector, whereas the in-core values are of course the thermal flux in the core. What one also can see is that the system becomes less critical, i.e. $\rho$ is negative, for increasing size. On the other hand a decrease of the core increases the reactivity and the system is super-critical. The out-of-phase behaviour is still present in the whole core in both cases.

When we increase the system size and use a more realistic displacement of 1 mm it is still possible to notice the out-of-phase behaviour but it is not appearing as distinctly as for the smaller system. The reactivity change is really small, $\sim 10^{-15}$, this means that the point reactor term is almost negligible, as expected, because a small change to a large system is

![Graph](image-url)
not affecting the reactivity that much. As one can see in Fig. 62 the out-of-phase (shaded areas) behaviour appears in the middle for both 1 mm larger and 1 mm smaller core.

Thus, by comparing the ex-core position (detector) with in-core positions (detectors) close to the centre of the reactor it should be possible to detect shell mode vibrations. In fact it should be possible by using only in-core positions (detectors), one close to the centre and one somewhere between the centre and the core-barrel. The perturbed thermal flux, \( \delta \phi_2(x) \), in Fig. 63, clearly illustrates the phase behaviour. One can also see that the ex-core flux is increasing with increasing size and decreasing with decreasing size.

For a closer look at the interesting area around the boundary between the core and the reflector Fig. 64 displays this part of Fig. 62. Here it is possible to see that a position close to the core-barrel also shows an out-of-phase behaviour with respect to an ex-core position. The advantage with this position compared to the one in the centre of the reactor is that here the change of the flux is much greater (see also Fig. 63) so it would be easier to detect...
vibrations. But on the other hand it is impossible to have a detector at this position in a real core.

One other way of describing the vibration would be to not change the cross sections and diffusion constants, instead just change the size of the core. Then the reactivity change is much larger, meaning a contribution from the point reactor term and that removes the out-of-phase behaviour from the centre of the core, Fig. 65

One way of describing the vibration would be to not change the cross sections and diffusion constants, instead just change the size of the core. Then the reactivity change is much larger, meaning a contribution from the point reactor term and that removes the out-of-phase behaviour from the centre of the core, Fig. 65

**Fig. 64.** This figure is a zoom in, of the core boundary region, of Fig. 62. The dashed vertical line is the core boundary before the movement and the dotted one is after the movement.

**Fig. 65.** a) displays the static and perturbed flux for a 1 mm outward movement of the core boundary without any change of the cross sections and the diffusion constants. b) displays the thermal noise $\phi(x)$.

### 4.7. Conclusion

We have solved the 2-group diffusion equation in a 2-region 1-D system and simulated a core barrel vibration by assuming a larger (smaller) core with higher (lower) density, change of the cross sections and diffusion constants. Then we applied reactor dynamics to
see if it is possible to get an out-of-phase behaviour between ex-core and in-core positions. The results show that there is an out-of-phase behaviour between ex-core positions (detectors) and positions around the centre of the core and also positions (detectors) close to the boundary between the core and the reflector. The out of phase behaviour in the centre if the core disappears if the density is kept constant. It is also shown that the reactivity change of the system is almost negligible meaning that the point reactor term of the perturbation is small. Thus, it is the local part of the noise that is dominating.

5. ANALYSIS OF THE LPRM MEASUREMENTS PERFORMED IN RINGHALS-1 ON SEPTEMBER 3-6, 2002 FOR THE INVESTIGATION OF DETECTOR STRING IMPACTING

5.1. Introduction

This section describes results of an analysis of the measurement already mentioned in Section 2, with the purpose of investigating detector tube impacting. As already mentioned, four measurements were made between 3-6 September 2002, at the Ringhals-1 BWR. The first three measurements were taken during start-up at a reduced core flow (about 4000 kg/s) and a reduced power level (about 70%), where the probability of core instability is the largest. Investigation of the possibility of local or global instabilities was the subject of Section 2. The last of the four measurements, measurement \( d \), was performed at full power and full core flow with a sampling frequency of 12.5 Hz. It is at full power and core flow when the probability of detector string impacting is the highest, hence this measurement is suitable for an analysis of impacting. Already at the time of the measurements it was clear to Ringhals personnel that some LPRM signals indicated the presence of detector string vibrations, with a considerable likelihood of impacting. We have performed an analysis of LPRM signals from all 36 positions, from two axial levels (levels 2 and 4 where the latter is the uppermost level) in order to quantify the probability and severity of detector tube impacting. The results of the analysis of measurement \( d \) are reported in this Section.

The analysis was made by two different methods. The first is the conventional method of investigating the auto-spectra of the signals and the coherence and phase between the two detectors within the same tube. The principles of this method were elaborated and applied to the detector tube vibrations in Barsebäck-1 and reported in [22]. These principles are also summarised in the next subsection.

The second method is based on calculating the so-called impacting rate by a wavelet-based procedure developed at our Department recently and tested on Oskarshamn data (Refs [23]-[25]). In contrast to the conventional method, this latter procedure is fully algorithmic and requires no knowledge of a reference measurement without impacting. It only requires the specification of a cutoff frequency such both the detector tube vibrations as well as the hypothetic transient and intermittent fuel assembly vibrations lie below this frequency. Then the rms value of the autospectrum is used for the determination of the background noise, which is necessary for the selection of the correct alarm level.

Unfortunately, the measurement is not optimal for the wavelet analysis, because the sampling frequency of 12.5 Hz is too low. This sampling frequency means that the spectral quantities can be calculated up to the Nyquist frequency of 6.25 Hz, which is just above the
resonance frequency of the detector string vibrations, and with all likelihood it is below the (unknown) impact induced fuel box vibrations. The essence of the wavelet based method is the detection of the spikes in the signal due to the transient fuel box vibrations. In future measurement it is therefore essential to use a higher sampling frequency.

The spectral analysis is not affected by the low sampling frequency, but it gives a more qualitative result. However, the combined application of the two methods (frequency analysis and wavelets) makes it possible to make a quantification of the impacting probabilities and severity with a rather good confidence. Accordingly, we have identified strings with a high likelihood of impacting, strings with some likelihood of impacting and some others with little or no impacting or even vibrations. Details of the analysis are given in the following pages.

5.2. Principles of the spectral analysis method

As described in [22], the following properties can be used for identifying the impacting status of the strings from the LPRM auto- and cross-spectra (coherence and phase).

No vibrations:
• smooth structure of the APSD and the coherence, no peaks;
• linear phase between the two detectors in the same string, showing the time delay of the signal (bubble transit time).

Vibrations but no impacting:
• a single narrow peak in the APSD and the coherence; linear phase distorted in the narrow frequency range of vibrations, the phase tends to bee zero in a narrow range.

Indications of impacting:
• a broad peak in the APSD
• several peaks, mainly a second peak at the double frequency of the first one
• phase curve distorted and zero over a large frequency range.

5.3. Analysis of the measurements

5.3.1. The vibration-free case

It is useful to start the analysis with a string where no vibrations at all are suspected. Such a signal is represented by LPRM 1 (Fig. 66), and it can be used as a reference of the vibration-free case.

As it is seen from the Figure, there is no peak in the APSD, rather it has a smooth structure. The phase behaviour is linear up to 4 Hz. The 180 crossing of the phase is at the same frequency as the first minimum (zero) of the coherence, and the zero crossing of the linear part of the phase curve appears at a frequency (~3 Hz) corresponding both the second maximum of the coherence and also to the inverse of the bubble transit time, about 0.3 sec, between the two detectors. The Amplitude Probability Distribution (APD) is also a standard Gaussian distribution. The IR number, impacting rate index, provided from the wavelet-based analysis and shown in the sub-figure containing the APSD, was calculated with the biorthogonal wavelet bior3.1. This number can then be compared with the IR values from the other strings.
5.3.2. Cases of heavy impacting

The most probably (and heavily) impacting strings were found to be LPRM 9, LPRM 10 and LPRM 22. In the ASPD of LPRM 22 (Fig. 67), at level 2 one observes one peak around 1.4 Hz, another one at the double frequency and a third one around 4.2 Hz. The two first peaks are also visible in level 4. The peak at the double frequency may be due to a large vibration amplitude at 1.4 Hz with the vibration taking place in a curved flux. There is also a high coherence between the two detectors at these three frequencies. The phase deviates from linear and is around zero everywhere, in particular at the vibration frequencies. The IR factor is larger by a factor 5-8 than in the previous case, and is the highest among all strings. In addition, the APD of the LPRM at level 2 shows two peaks instead of the normal gaussian distribution and a large standard deviation. All these facts indicate a vibrating detector tube which is impacting on the fuel box, with a rather high confidence.
Another case with a heavy impacting is that of LPRM 9 (Fig. 68). Signals from this string showed a large peak in the APSD at 2 Hz already in measurement $b$, i.e. taken at reduced core flow (about 4000 kg/s) and a reduced power level (about 70%). The peak is present also in measurement $d$, but at the lower frequency of 1.5 Hz, and it is much broader. The peak broadening indicates that in measurement $b$ the string was vibrating without impacting, whereas in measurement $d$ it was impacting. There is also a peak at the double frequency. We also notice the zero phase of the CPSD, typically for a vibrating and impacting detector and the high coherence around 1.5 Hz. Again the IR number is higher than for most other signals.

Fig. 67. LPRM 22, 2002-09-06 07:55-07:06, Most probable impacting
Fig. 68. LPRM 9, 2002-09-06 07:55-07:06, Most probable impacting
The last case of most likely heavily impacting string is that of LPRM 10 (Fig. 69). There is a peak in the APSD at around 1.8 Hz and a zero phase and high coherence at that frequency. The wavelet analysis also yielded a high IR number. The final judgement is that this detector is most likely impacting on the fuel assemblies.

5.3.3. Cases of mild vibrations without low probability of impacting

LPRMs 4, 8, 11, 12, 16, 24, 30 and 33 show also signs of vibration, and the possibility of their impacting cannot be fully excluded. However, the amplitude of the vibrations and the probability and the severity of impacting is significantly less than in the case of LPRMs 9, 10 and 22. As an example, LPRM 8 (Fig. 70), shows zero phase and also high coherence around the 2.1 Hz peak, but the peak is not as broad as for the signals of the three heavily impacting LPRMs.
impacting LPRMs. This indicates that the string may impact but the possibility is smaller than for LPRMs 9, 10 and 22.

At the “lower end” of this spectrum one can find strings with signs of vibrations but with no impacting. One such example of a vibrating detector without impacting is LPRM 15 (Fig. 71). It has a peak in the APSD at 1.6 Hz and the coherence at this frequency is high but the peak is rather thin. In the CSPD the phase is almost linear but something is happening at 1.6 Hz. So LPRM 15 looks like LPRM 1 except for these things. The thin peak in the coherence indicates that it is a clean vibration (no impacting) at 1.6 Hz.

5.4. Summary and conclusions

The results of the analysis can be summarized as follows. The strings with highest probability and severity of impacting are LPRMs 9, 10 and 22. In the next, milder category one can put LPRMs 4, 8, 16 and 24. The detectors with the least chance of impacting, out of the suspected ones, are LPRM 11, 12, 30 and 33. These conclusions are also summarised in

Fig. 70. LPRM 8, 2002-09-06 07:55-07:06, Maybe impacting
Table 3. Ranking of the most probable impacting detectors

<table>
<thead>
<tr>
<th>LPRM</th>
<th>Impacting status</th>
</tr>
</thead>
<tbody>
<tr>
<td>9, 10 and 22</td>
<td>most likely impacting</td>
</tr>
<tr>
<td>4, 8, 16 and 24</td>
<td>maybe impacting</td>
</tr>
<tr>
<td>11, 12, 30 and 33</td>
<td>some, but small, chance of impact</td>
</tr>
</tbody>
</table>

For sake of completeness, and for supporting the judgement made in the analysis, in the following pages all APSDs, coherence and phase from all LPRM signals in measurement \( d \), are displayed.
6. PROPOSAL FOR STAGE 9

The following items are proposed for Stage 9:

1. Further development, as well as test and application of the noise analysis based method for the determination of the moderator temperature coefficient.

2. Further development and test of advanced analysis methods for diagnostics of core barrel vibrations with the use of both in-core and ex-core neutron noise.

3. Analysis of a measurement taken in R1 during start-up as well as at full power, for investigation of diagnostics of local/regional stability and detector string impacting.

4. Investigation of the conditions and possibilities of a so-called “diagnostics centre”.

A more detailed contract proposal is sent to Ringhals regarding Stage 9.

7. ACKNOWLEDGEMENT

Stage 8 of the project was performed by funding from Ringhals Vattenfall AB, contract No. 531970-003. Contact person from Ringhals was Tekn. Lic. Tell Andersson. We acknowledge the receipt of the measured signals of Ringhals-2 PWR (measurement performed in March 2003) and of Ringhals-1 BWR (measurement performed in September 2002).

8. REFERENCES


Chalmers University of Technology, Gothenburg, Sweden


Research with Artificial Intelligence, Editors: Da Ruan and Paolo F. Fantoni, Springer, Physica Verlag XIV, pp 157 - 174
